Population Growth & Decline

Quiz

What is the difference between an open and a closed population?

(A) An open populations is affected by only reproduction, whereas a closed population is affected by reproduction and mortality

(B) An open population is subject to immigration and emigration, whereas a closed population is not

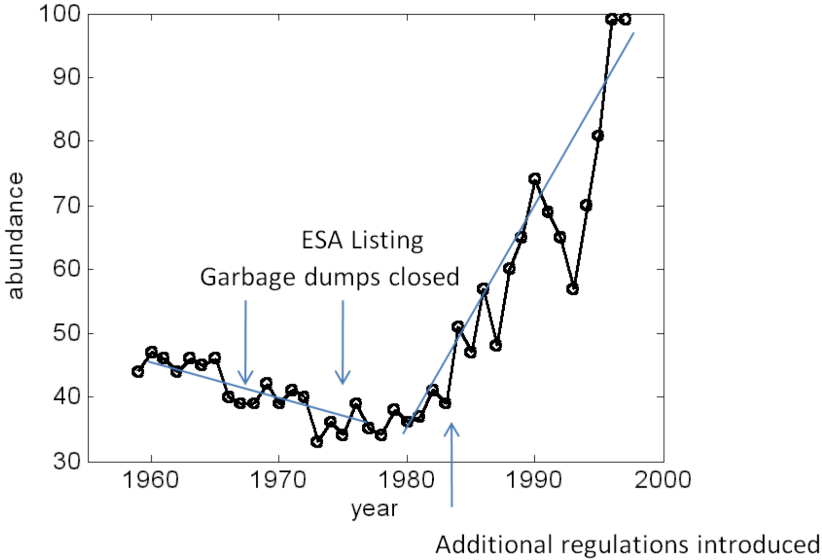
(C) Closed populations never experience reproduction, whereas open populations do

(D) A closed population is at it's maximum size, whereas an open population has room to grow

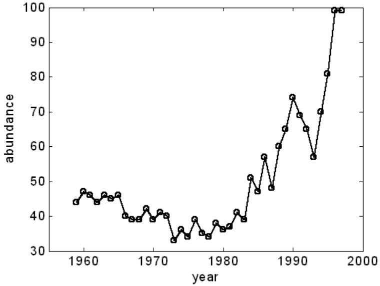
Grizzly Bear

Ursus arctos horribilis

Recovery of Grizzly Bear population in Greater Yellowstone ecosystem



Questions about growth and decline



(1) How much change in mortality was required to shift the balance from decline to growth?

(2) If interventions had not been undertaken, would the population have gone extinct? When?

(3) Is the population safe now? What is the chance that it might still go extinct?

What causes growth and decline?

Four key demographic processes:

Closed populations include only

(1) Reproduction (+)

(2) Mortality (-)

Open populations are also subject to

(3) Immigration (+)

(4) Emigration (-)

Modeling demography

The fundamental equation of population ecology:

$$\Delta N = N_{t+1} - N_t = B_N - D_N + I_N - E_N \qquad (1)$$

Three additional assumptions

(1) Population is closed

(2) No heterogeneity (differences among individuals)

(3) No density dependence ($B_N = BN_t$; $D_N = DN_t$)

$$\Delta N = N_{t+1} - N_t = BN_t - DN_t + \cancel{I_N} - \cancel{E_N} \qquad (2)$$

Modeling demography

Starting with

$$\Delta N = N_{t+1} - N_t = BN_t - DN_t + \cancel{I_N} - \cancel{E_N} \tag{3}$$

We can rearrange to a <u>recursive formula</u>:

$$N_{t+1} = BN_t - DN_t + N_t \tag{4}$$

Defining $\lambda = B - D + 1$, we simplify to

$$N_{t+1} = \lambda N_t \tag{5}$$

The symbol λ is the Greek letter lambda and denotes the <u>discrete time population growth rate</u> or <u>reproductive multiplier</u>

Generalizing to more than one time step

Starting with

$$N_{t+1} = (BN_t - DN_t + 1)\, N_t = \lambda N_t \tag{6}$$

We also have

$$N_{t+2} = (BN_t - DN_t + 1)\, N_{t+1} = \lambda N_{t+1} \tag{7}$$

Inserting the first equation into the second, we have

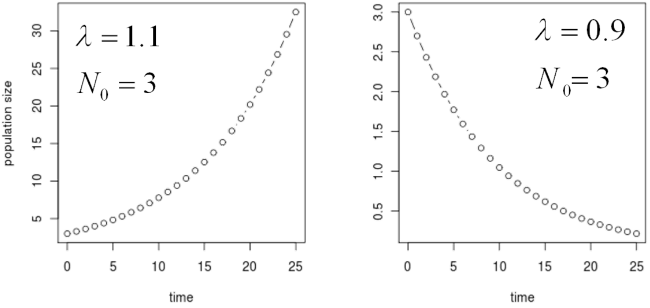
$$N_{t+2} = \lambda\lambda N_t = \lambda^2 N_t \tag{8}$$

In general,

$$N_t = \lambda^t N_0 \tag{9}$$

Geometric growth and decline

What does the equation $N_t = \lambda^t N_0$ look like?



Estimating λ from data

$$
\begin{aligned}
N_t &= \lambda^t N_0 \\
N_t/N_0 &= \lambda^t \\
\log\left(N_t/N_0\right) &= \log\left(\lambda^t\right) \\
\log\left(N_t/N_0\right) &= t\log\lambda \\
\frac{\log\left(N_t/N_0\right)}{t} &= \log\lambda \\
e^{\frac{\log\left(N_t/N_0\right)}{t}} &= \lambda
\end{aligned}
\tag{10}
$$

Problem

Given that the Yellowstone Grizzly population was 44 in 1959 and 34 in 1975, what was the average annual reproductive ratio during this period?

Challenge Problems

Problem

Given $N_0 = 3$, $N_1 = 4$, $N_2 = 6$ and using our equation we have an estimate of $\lambda = 1.414$, but this discards some of the information. What information is discarded? How can this information be used? What is the new estimate?

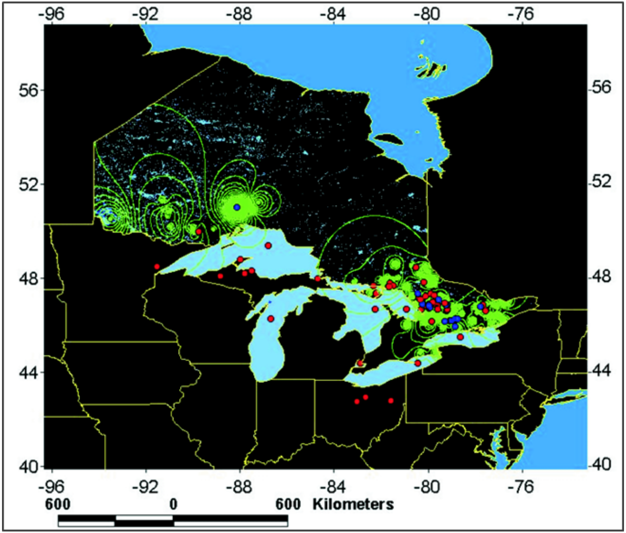
Problem

At what values of λ and/or N is the system at equilibrium?

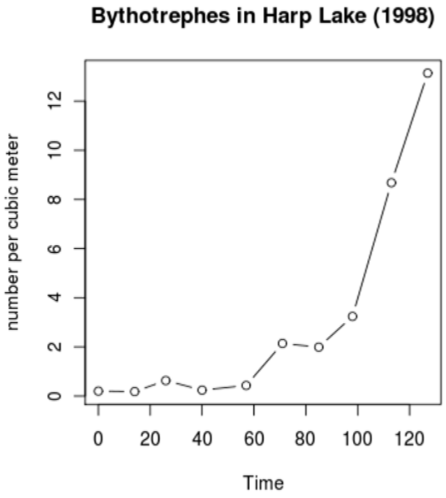
Growth and decline of Spiny water flea

Bythotrephes longimanus

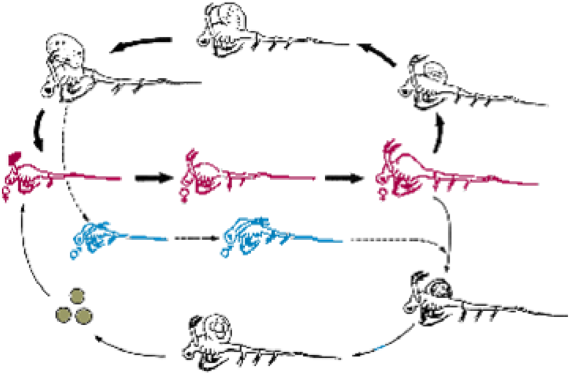
Growth and decline of Spiny water flea



Spiny water flea invasion of Harp Lake, Ontario



Life cycle of Spiny water flea is continuous



Instantaneous population growth

An alternative form of the fundamental equation of population ecology

$$\frac{dN}{dt} = bN - dN + i - e \tag{11}$$

For now, we assume a closed population and define the intrinsic rate of increase: $r = b - d$

$$\frac{dN}{dt} = bN - dN = (b - d)N = rN \tag{12}$$

Solution

$$
\begin{aligned}
\frac{dN}{dt} &= rN & & \\
\frac{1}{N}dN &= rdt & & \text{Separation of variables} \\
\int \frac{1}{N}dN &= \int rdt & & \text{Integrate} \\
\log N &= rt + C & & \text{General solution} \\
e^{\log N} &= e^{rt+C} & & \text{Algebra} \\
N &= e^{rt}e^{C} & & \\
N &= Ae^{rt} & & \text{What is } A\text{?}
\end{aligned}
$$

Solution

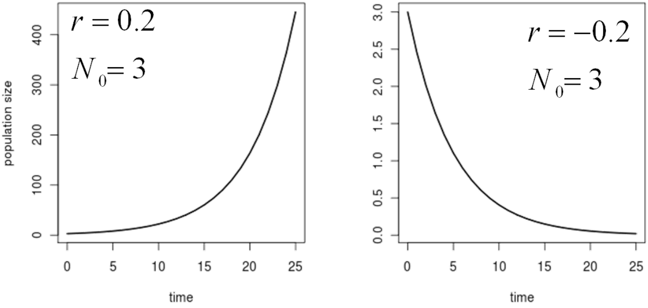
$$N = Ae^{rt}$$
$$N_0 = Ae^{r(0)}$$
$$N_0 = A$$

So...

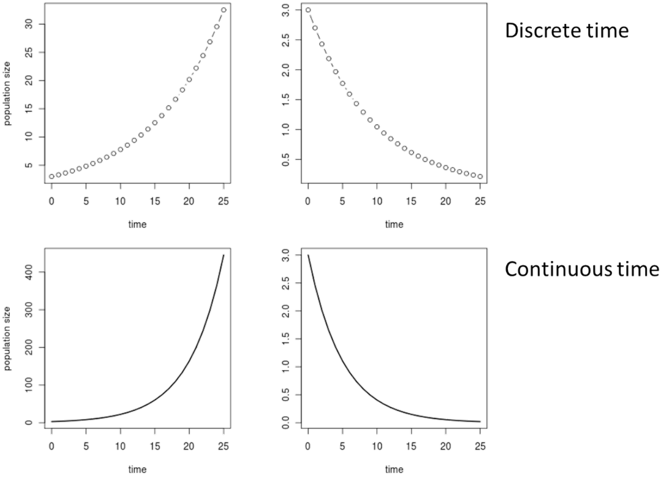
$$N_t = N_0 e^{rt}$$ $\leftarrow$ Remember this

Exponential growth and decline

What does the equation $N_t = N_0 e^{rt}$ look like?



A comparison of discrete- and continuous-time growth and decline



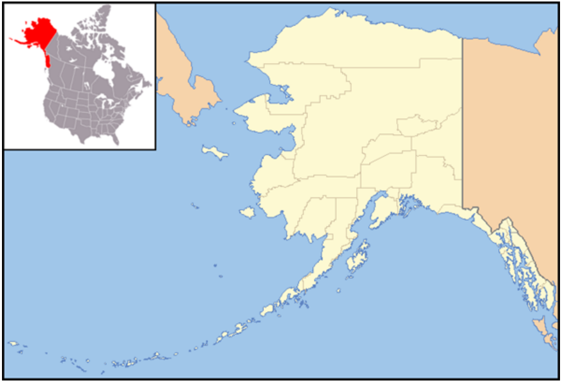
The relationship between discrete and continuous growth

$$\begin{aligned} \text{Solution for one year} \quad & N_t = N_0 e^{rt} \\ \text{Define } \lambda \text{ in terms of } r \quad & \lambda = e^{rt} = e^{r} \\ \text{Taking logarithms} \quad & \log \lambda = r \end{aligned}$$

A counter-example: Muskox on Nunivak Island

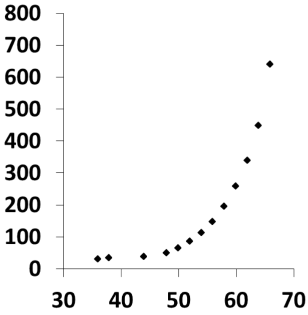
Ovibos moschatus

Reintroduction of Muskox on Nunivak Island



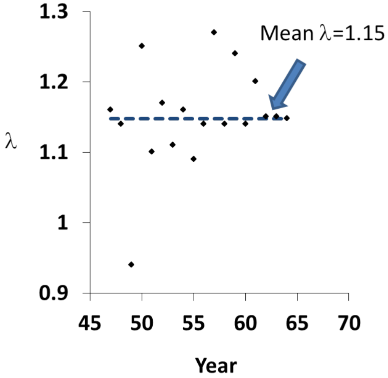
- Extirpated in the 19^{th} century
- 31 animals introduced in 1936 by USFWS
- $\approx$ 651 animals in 1970

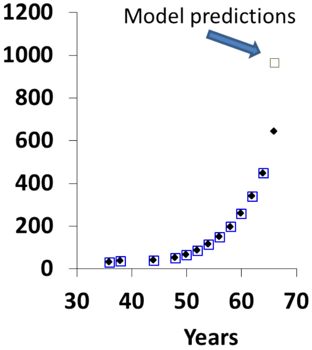
Reintroduction of Muskox on Nunivak Island



Time series of Muskox abundance on Nunivak Island
Source: USFWS

Reintroduction of Muskox on Nunivak Island





Annualized growth rates of Muskox abundance on Nunivak Island
Source: USFWS

Key points

- Fundamental equation (**know this**)
  - Open (including reproduction/mortality/immigration/emigration)
  - Closed (just reproduction/mortality)
- Change in population size depends on current size (N) and growth rate (r or λ)
- Three possible outcomes depending on growth parameter
  - $\lambda > 1$ or $r > 0$ (N increases)
  - $\lambda = 1$ or $r = 0$ (N remains constant)
  - $\lambda < 1$ or $r < 0$ (N decreases)
- Estimation - Student should be able to estimate λ or r from time series data
- Prediction - Student should be able to extrapolate a fit model to make a prediction about the population size at a future time

Homework

- Complete all six homework questions from Chapter 1
- Answers can be typed or scanned and submitted via Dropbox
- Due: Tuesday, August 28 at 5:00pm