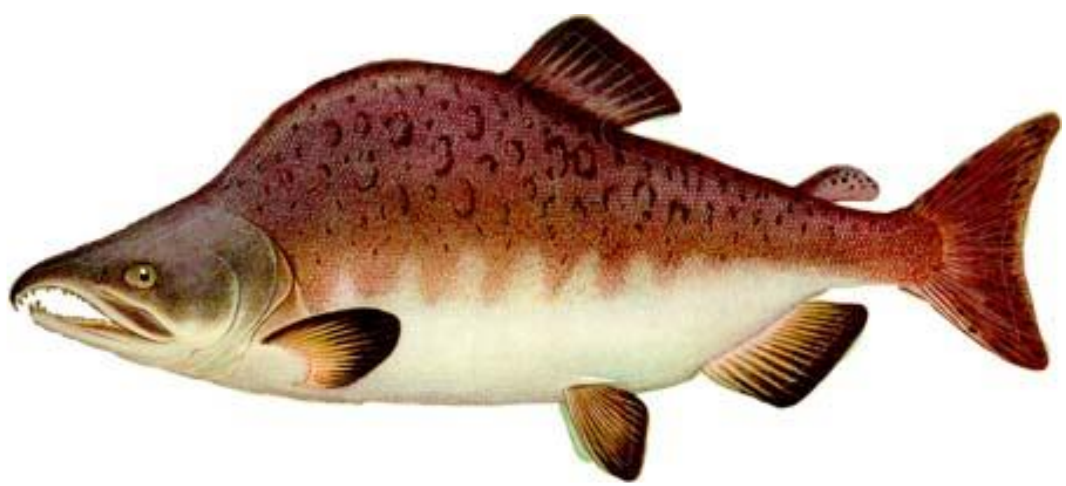
Complex Population Dynamics

Quiz

Which of the following are plausible reasons for the non-cycling of salmon populations?

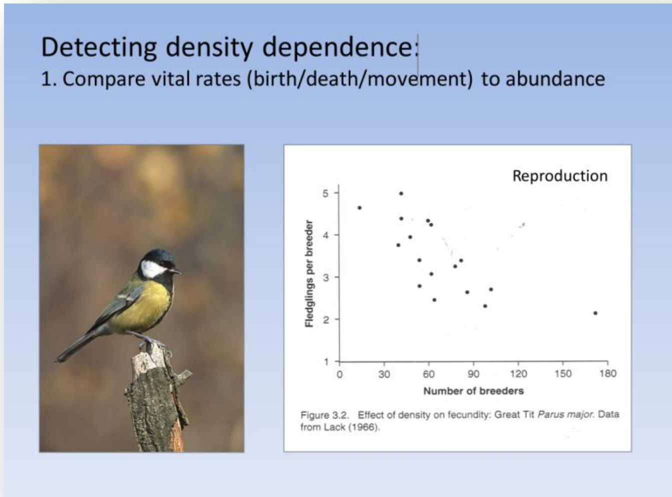
(a) Insufficient harvesting
(b) High harvesting
(c) Lack of severe density dependence
(d) Presence of severe density dependence

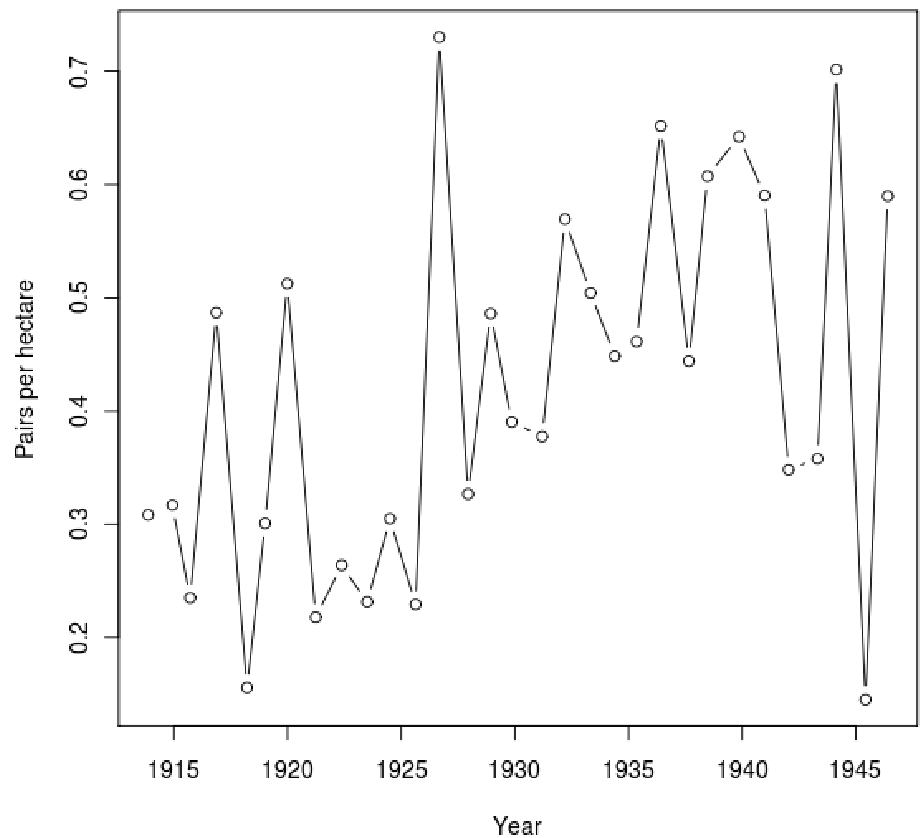


Concepts

- Population cycling
- Overcompensation
- Bifurcation
- Aperiodic dynamics
- Cobweb diagram
- Dynamics of age-structured populations

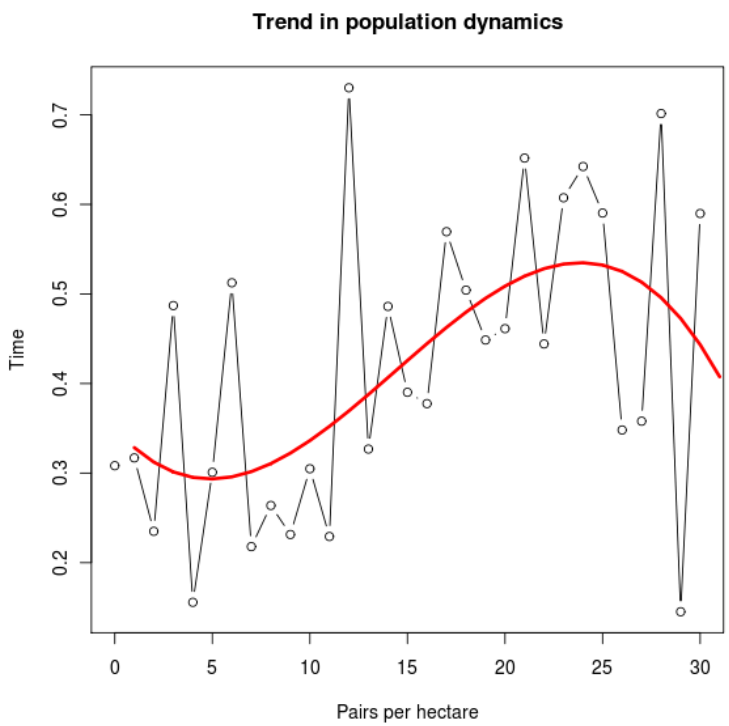
Population dynamics in *Parus*

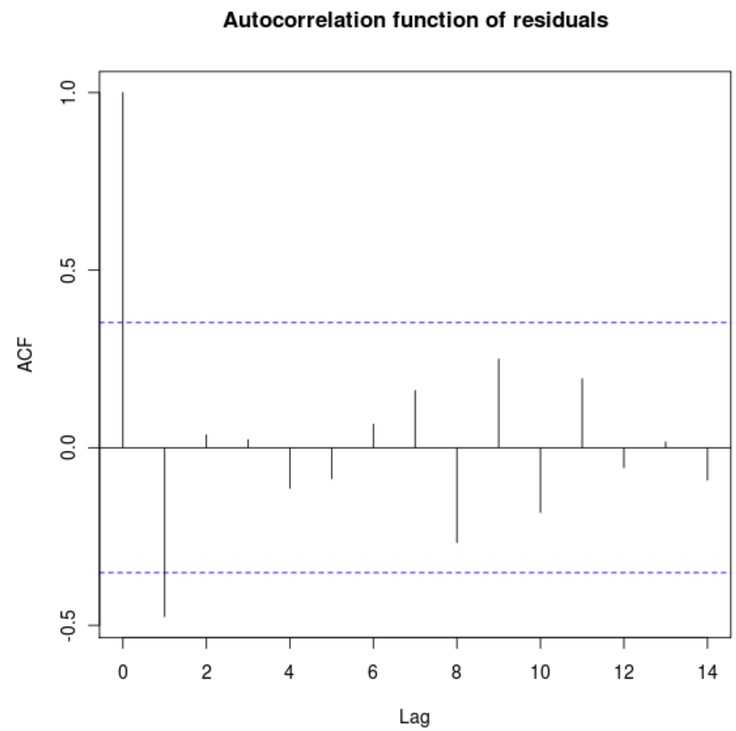




How would you describe these data?

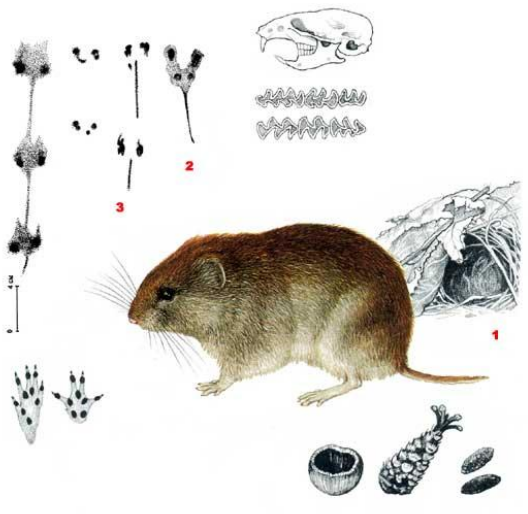
Is this population cycling?

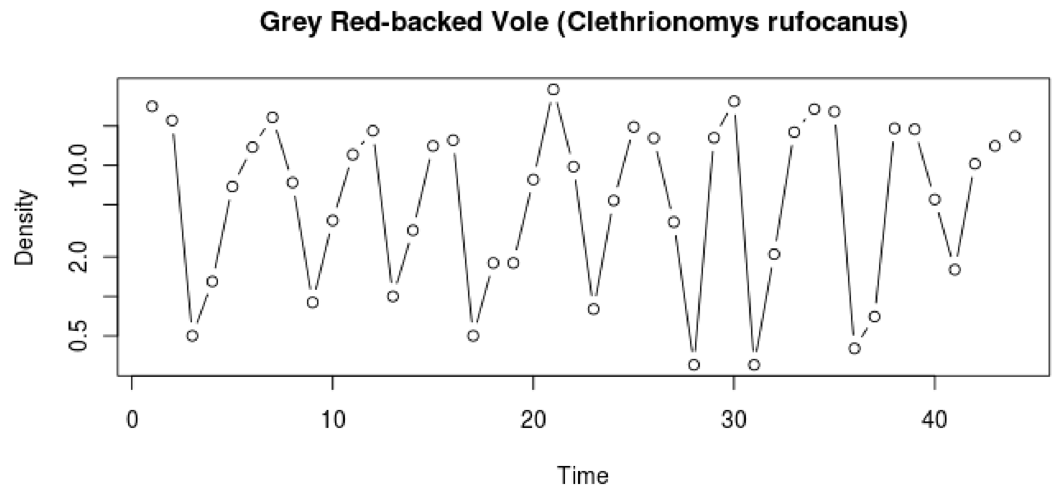


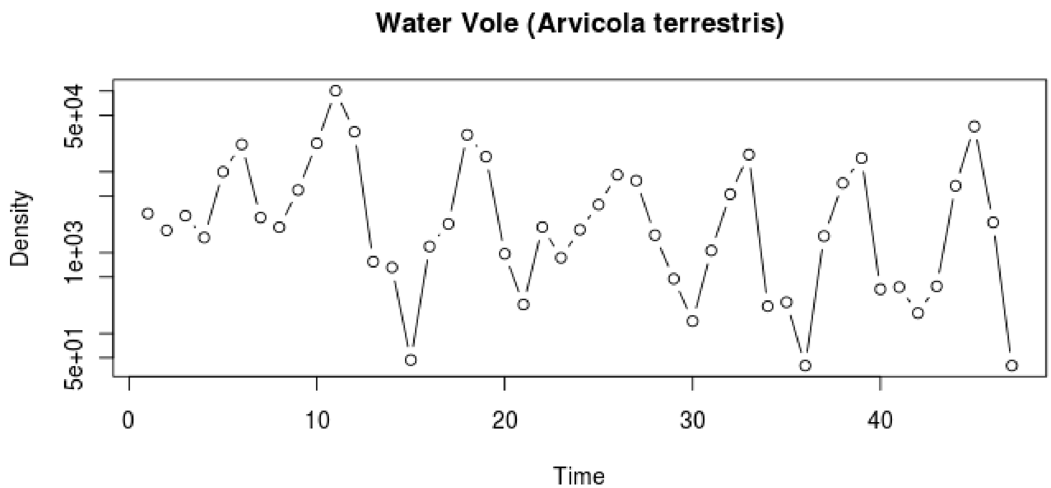


Parus major populations appear to be *cycling* around a moving baseline carrying capacity

Are there other classes of dynamical phenomena?







Turchin (2003) *Complex population dynamics.* Princeton University Press.

Frequency of Cyclical Dynamics

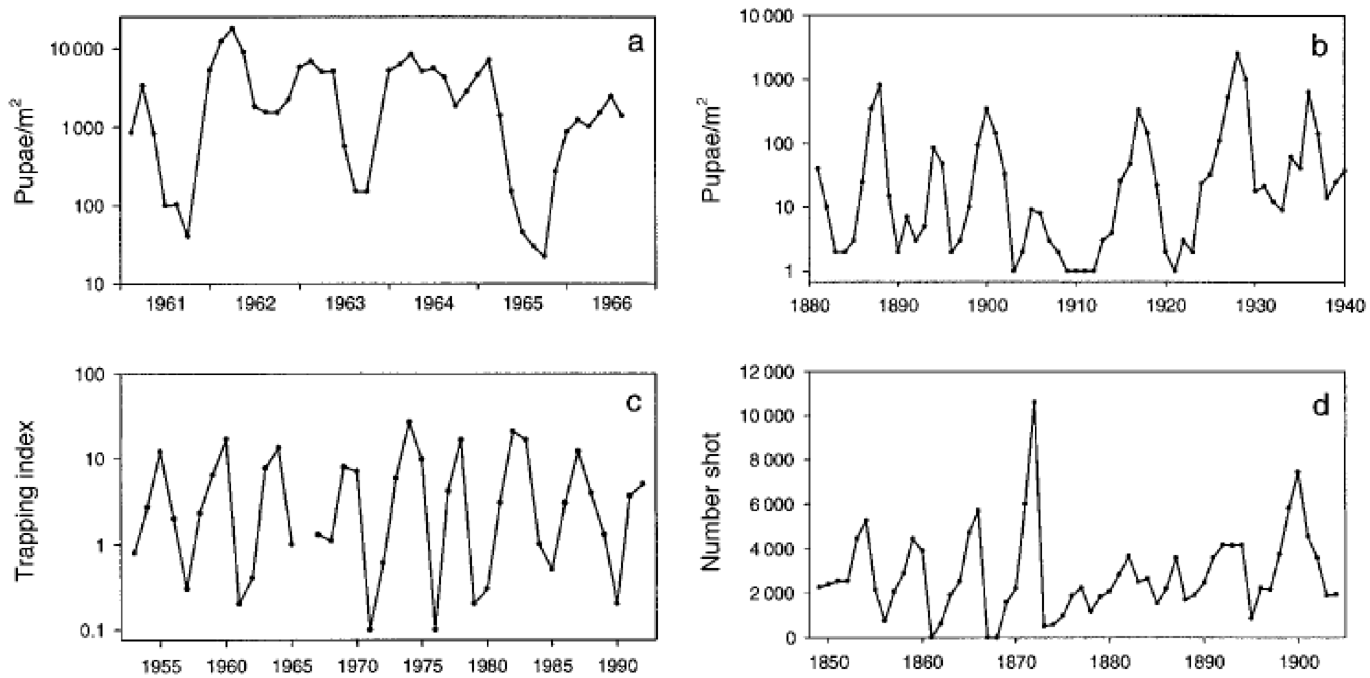


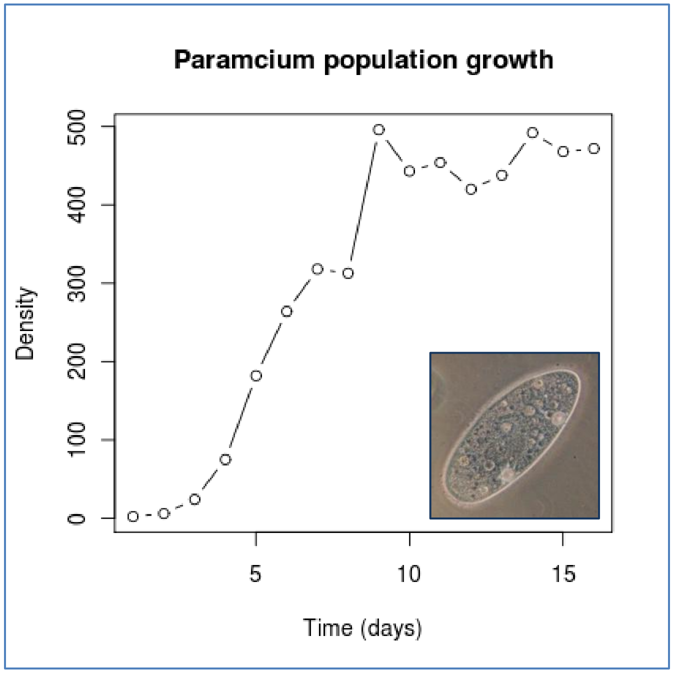
FIG. 1. Examples of cyclic population dynamics. (a) Coffee leaf-miners (*Leucoptera* spp.) at Lyamungu, Tanzania (Bigger 1973). (b) Pine looper (*Bupalus piniarius*) in Germany (Schwerdtfeger 1941). (c) Voles (*Microtus* and *Clethrionomys*) at Kilpisjärvi, northern Finland (Laine and Henttonen 1983, Hanski et al. 1993). (d) Red Grouse (*Lagopus lagopus scotius*) in Scotland (Middleton 1934).

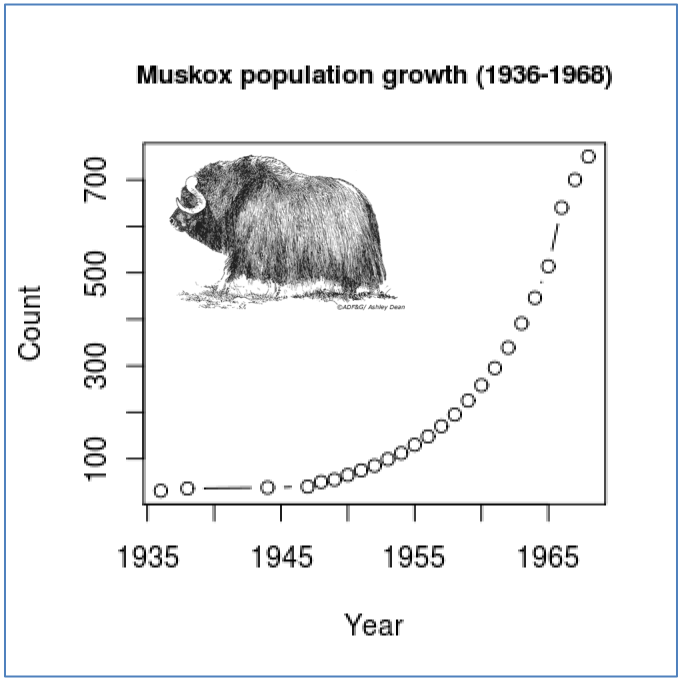
From Kendall et al. (1999) Ecology 80: 1789-1805.

30% of long term time series exhibit periodic oscillations

What causes populations to cycle?

- Classes of population dynamics
  - Exponential/geometric growth
  - Exponential/geometric decline
  - Equilibrium (carrying capacity)
  - Equilibrium (extinction)



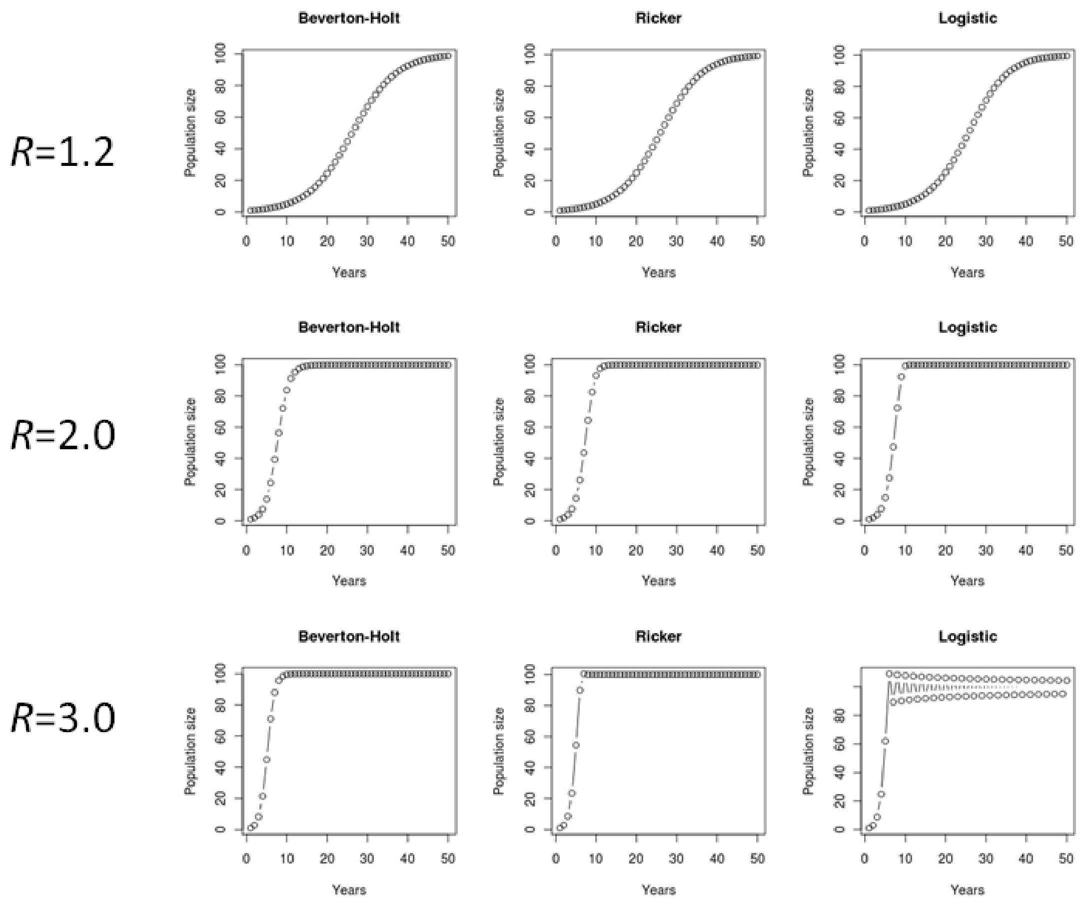


Density dependence redux

$$\lambda(N) = \frac{R}{1 + aN_t} \qquad N_{t+1} = N_t \frac{R}{1 + aN_t}$$

$$\lambda(N) = \lambda_0 e^{bN_t} \qquad N_{t+1} = N_t \lambda_0 e^{bN_t}$$

$$\lambda(N) = R(1 - N/K) \qquad N_{t+1} = N_t R(1 - N/K)$$

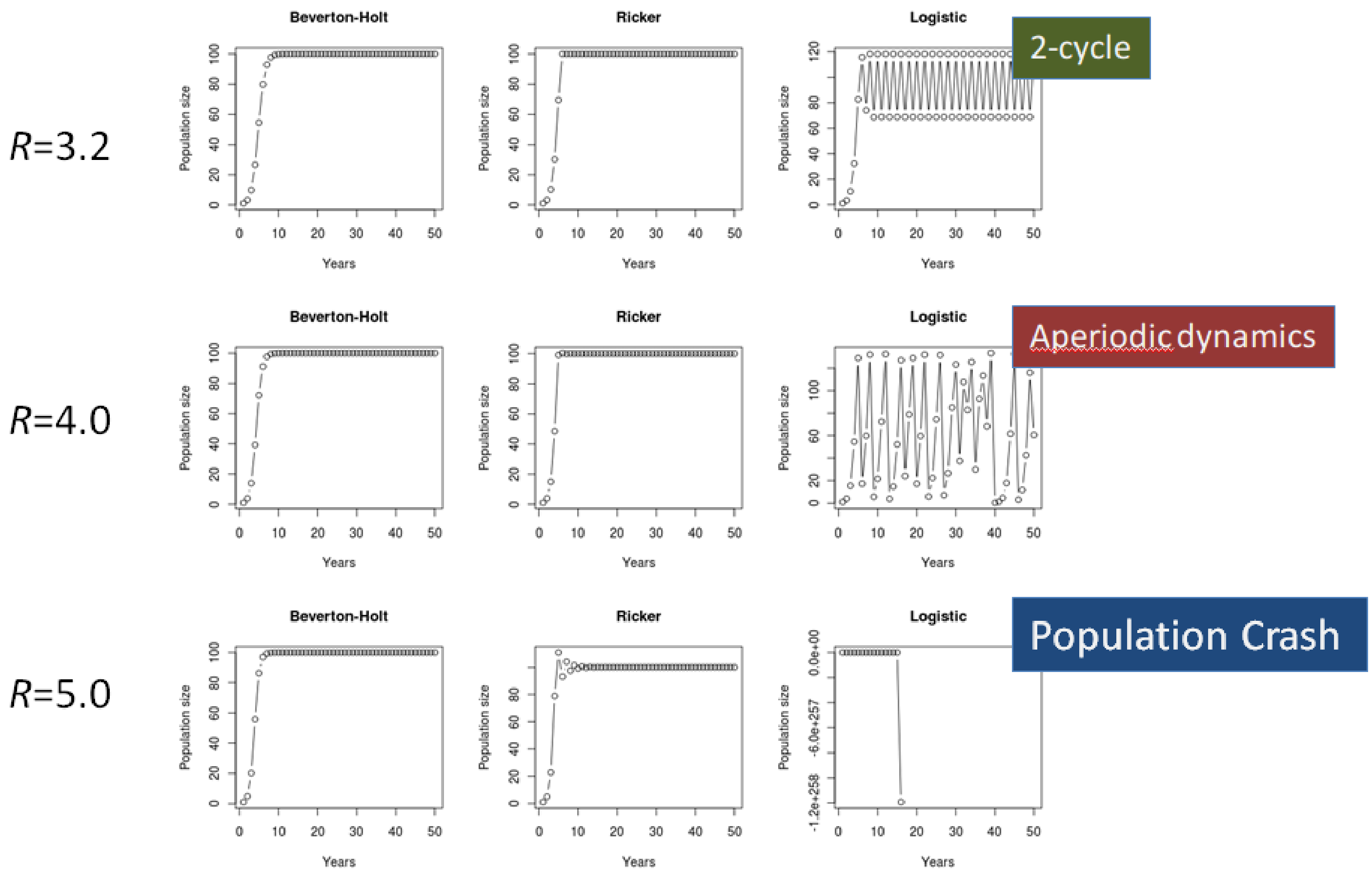


Overcompensation

Overcompensation occurs when populations with periodic reproduction “overshoot” carrying capacity

$$\lambda(N)=\frac{R}{1+aN_t} \qquad \lambda(N)=\lambda_0 e^{bN_t} \qquad \lambda(N)=R(1-N/K)$$

$$N_{t+1}=N_t\frac{R}{1+aN_t} \qquad N_{t+1}=N_t\lambda_0 e^{bN_t} \qquad N_{t+1}=N_tR(1-N/K)$$



Overcompensation

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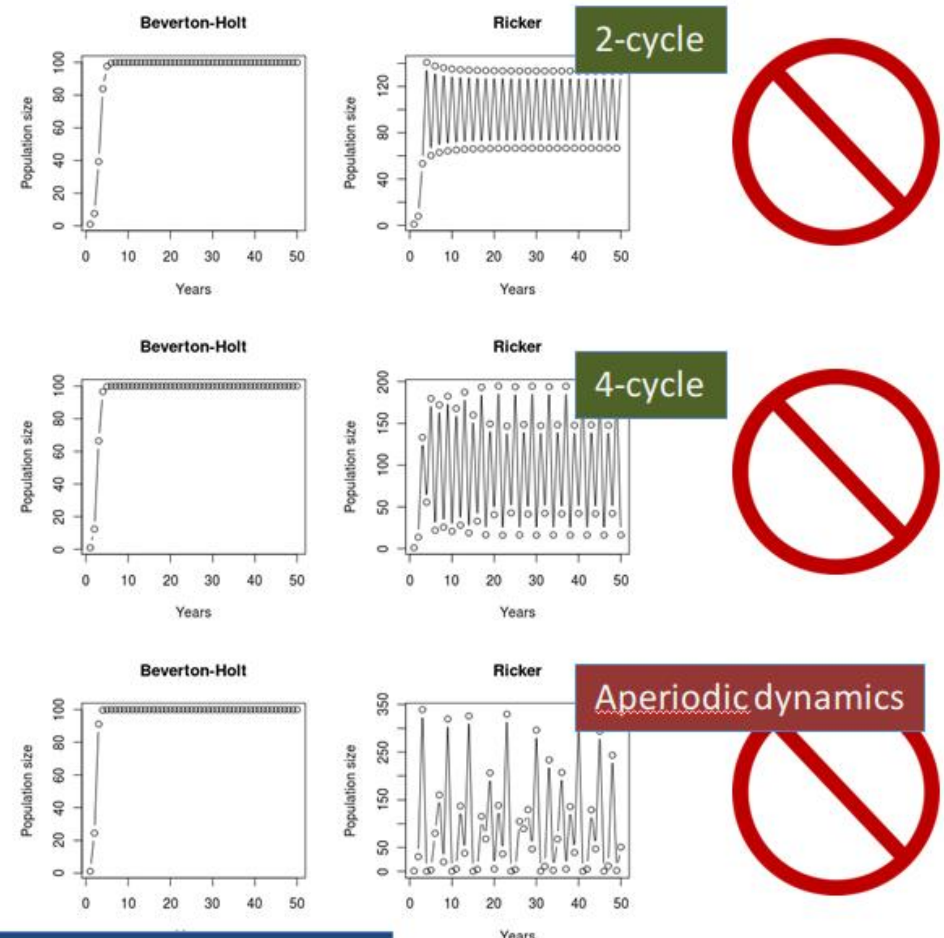
$$\lambda(N)=R(1-N/K) \qquad N_{t+1}=N_tR(1-N/K)$$

R=8.0

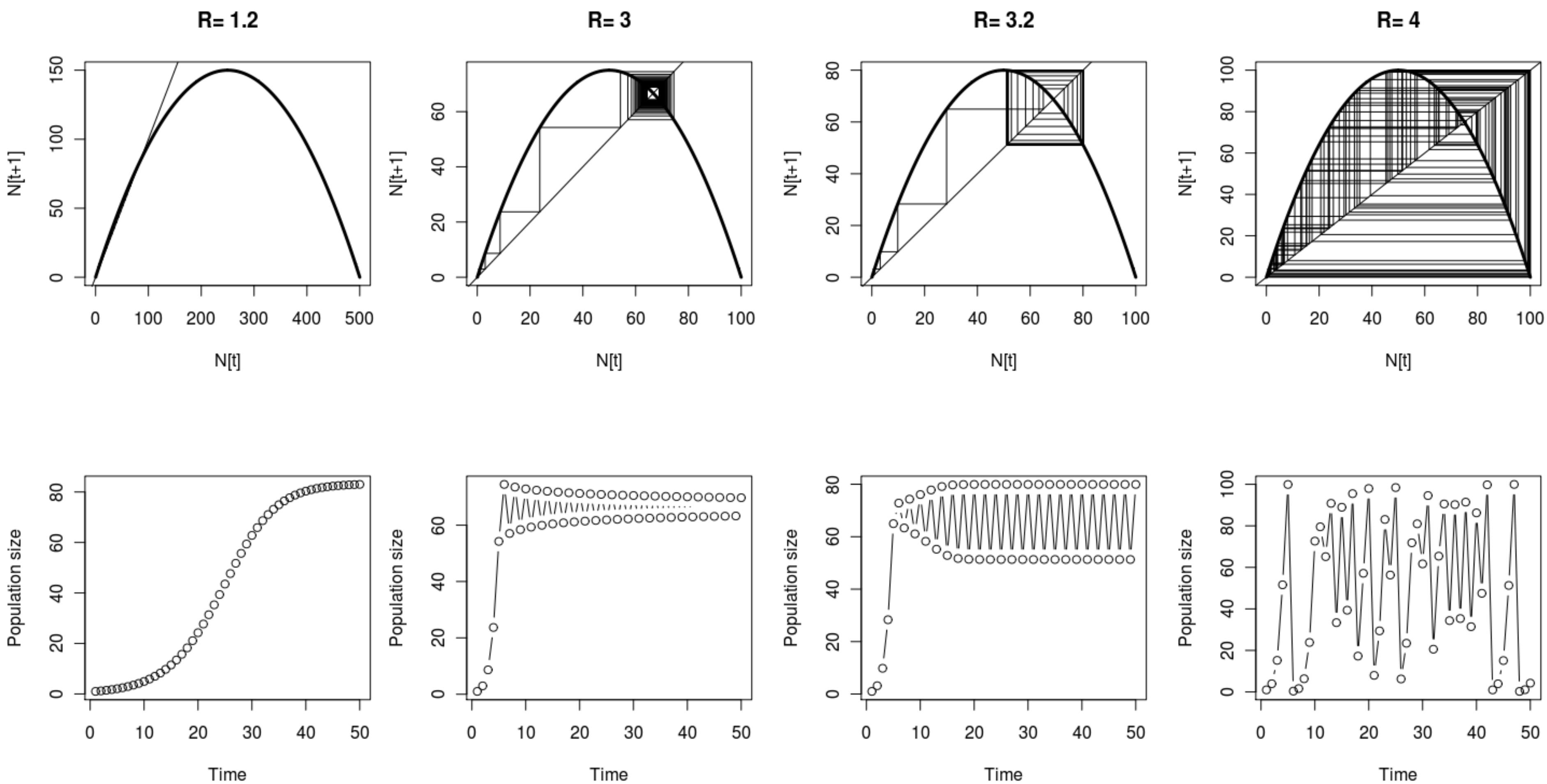
R=14.0

R=32.0



Beverton-Holt model always exhibits smooth dynamics

Cobweb diagrams for the discrete logistic model



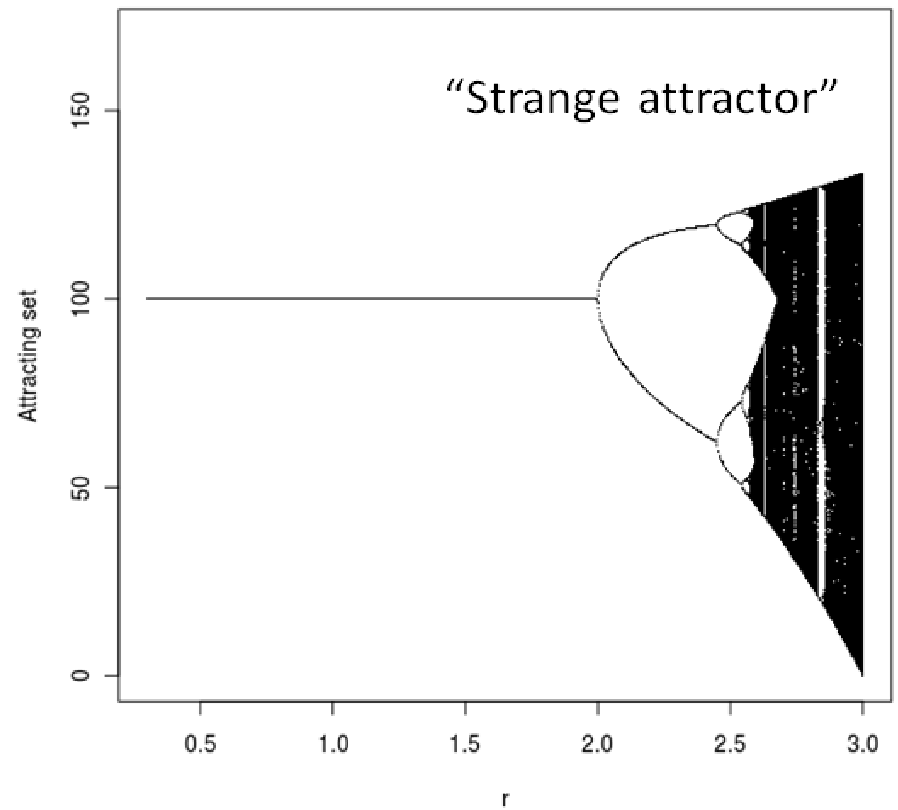
Bifurcation diagram

A "bifurcation" occurs when a smooth change in the parameters gives rise to a qualitative change in the long-run population dynamics

$$x_{t+1} = (1+r)x - \frac{rx^2}{k}$$

Bifurcation in 1-D maps
- saddle-node bifurcations
- transcritical bifurcations
- pitchfork bifurcations
- period-doubling bifurcations

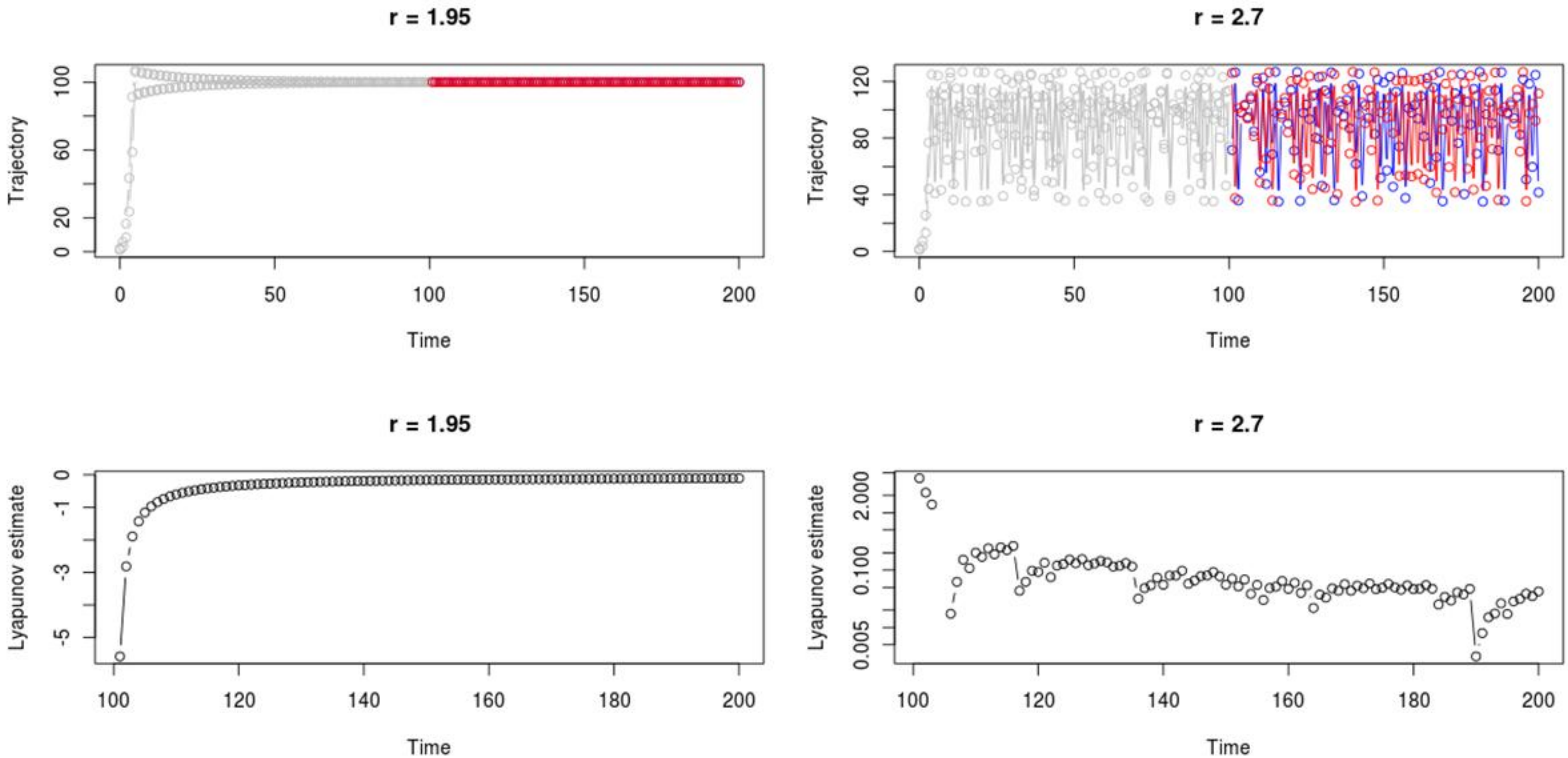
Aperiodic behavior
Frequency locking



Dynamical chaos

- Dynamical chaos is one kind of *aperiodic behavior* (population trajectory never repeats itself)
  - A candidate explanation for fluctuations in nature
- Characterized by *sensitivity to initial conditions*
- Fully deterministic, but nearly *indistinguishable from random noise*
- Characteristics predisposing a population to chaos include:
  - Annualized reproduction (or time lags)
  - High reproductive output
  - High nonlinearity

Sensitive dependence on initial conditions ("lyapunov exponent")



$$\lambda = \lim_{x \to \infty} \frac{1}{n} \sum_{i=0}^{n-1} \log\left(\left| \frac{df}{dx}(x_i) \right| \right)$$

Stability analysis of discrete time models

- Step 1. Let x^* be an equilibrium of the model. Define a new abstract quantity, u, to be a small perturbation away from x^*. We will ask the question, "Does u grow or shrink?"

$$u(t) = x(t) - x^*$$

- Step 2. Rewrite the dynamical system model using u

$$x_{t+1} = f(x_t)$$

becomes

$$x^* + u_{t+\Delta t} = f(x^* + u_t)$$

Stability analysis of discrete time models

Taylor's Theorem

$$f(x^*)+\frac{f'(x^*)}{1!}(x-x^*)^1+\frac{f''(x^*)}{2!}(x-x^*)^2+\frac{f'''(x^*)}{3!}(x-x^*)^3+\ldots$$

- Step 3. Linearize using Taylor's theorem

$$x^*+u_{t+\Delta t}=f(x^*)+uf'(x)_{x=x^*}$$

x* on l.h.s. cancels with f(x*) on r.h.s and we have an equation for the growth of our perturbation

$$u_{t+\Delta t}=uf'(x)_{x=x^*}$$

Rules for stability

- If f'(x)>1 : geometric growth
- If 0<f'(x)<1 : geometric decline
- If -1<f'(x)<0 : damped oscillations
- If f'(x)<-1 : divergent oscillations

Example with discrete logistic model

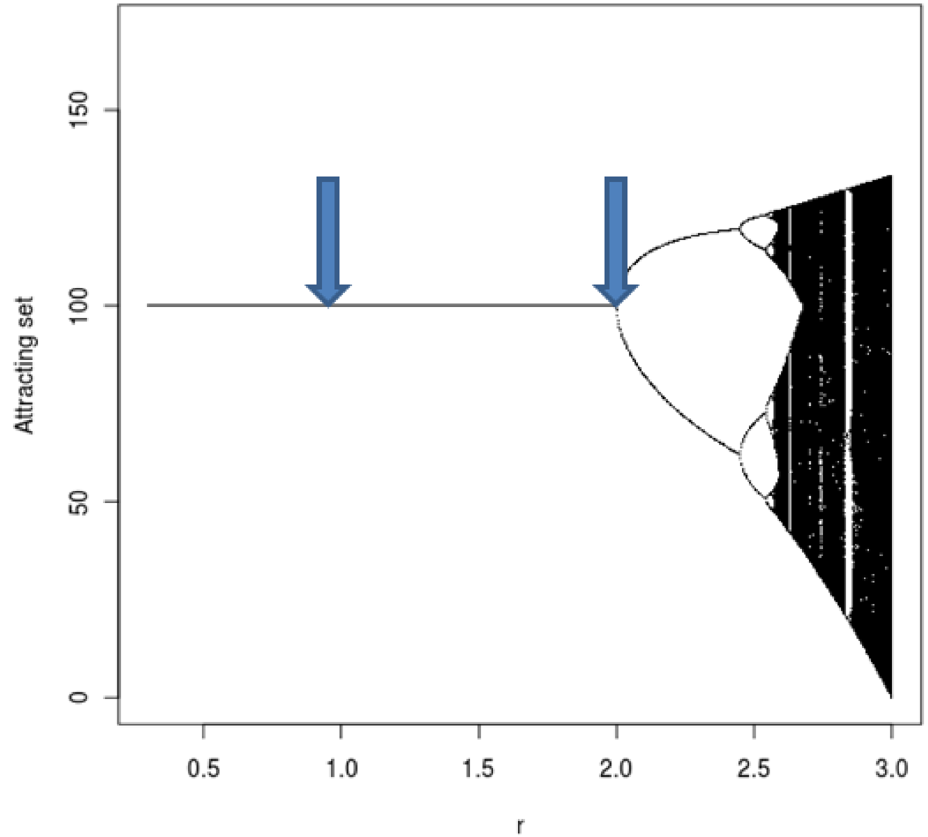
$$x_{t+1} = (1-r)x - \frac{rx^2}{k}$$

$$f'(x) = 1 + r - 2x\frac{r}{k}$$

$$f'(x=k) = 1 + r + 2x^*\frac{r}{x^*}$$

$$f'(x=k) = 1 + r - 2r$$

$$f'(x=k) = 1 - r$$



0<r<1 : geometric decline
1<r<2 : damped oscillations
R>2 : divergent oscillations

Bootstrap estimates of uncertainty by Falck et al. 2005