Extinction

Quiz

- In the simple birth-death stochastic model, if birth rate b > death rate d then the ultimate probability of extinction is:
- A) $P=0$
- B) $P=1$
- C) $0<P<1$

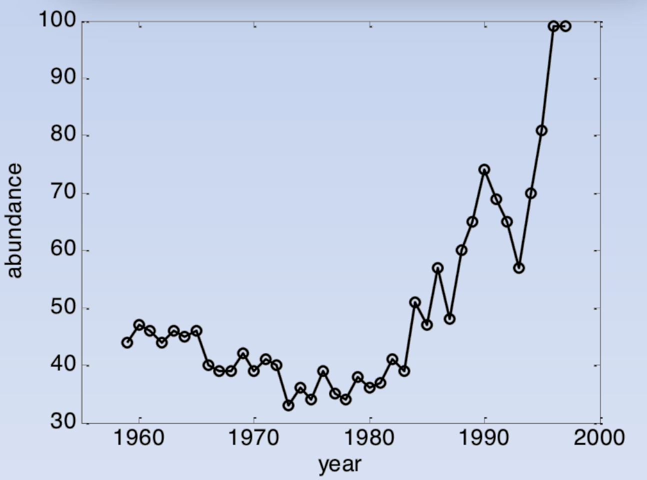
Key concepts

- Stochastic birth-death model
- Probability of ultimate (eventual) extinction
- Demographic stochasticity
- Square root scaling law
- Population viability
- IUCN criterion E

The stochastic process of extinction

- A problem with *deterministic* geometric models: For finite time the solution of these models implies finite population size

- How does population extinction occur?
- *Stochastic process*: a dynamical process in which part of the rate of change is determined by a *random variable*



The fundamental idealization: identical individuals

Exponential Growth:

Discrete
$$N_{t+1} = RN_t$$

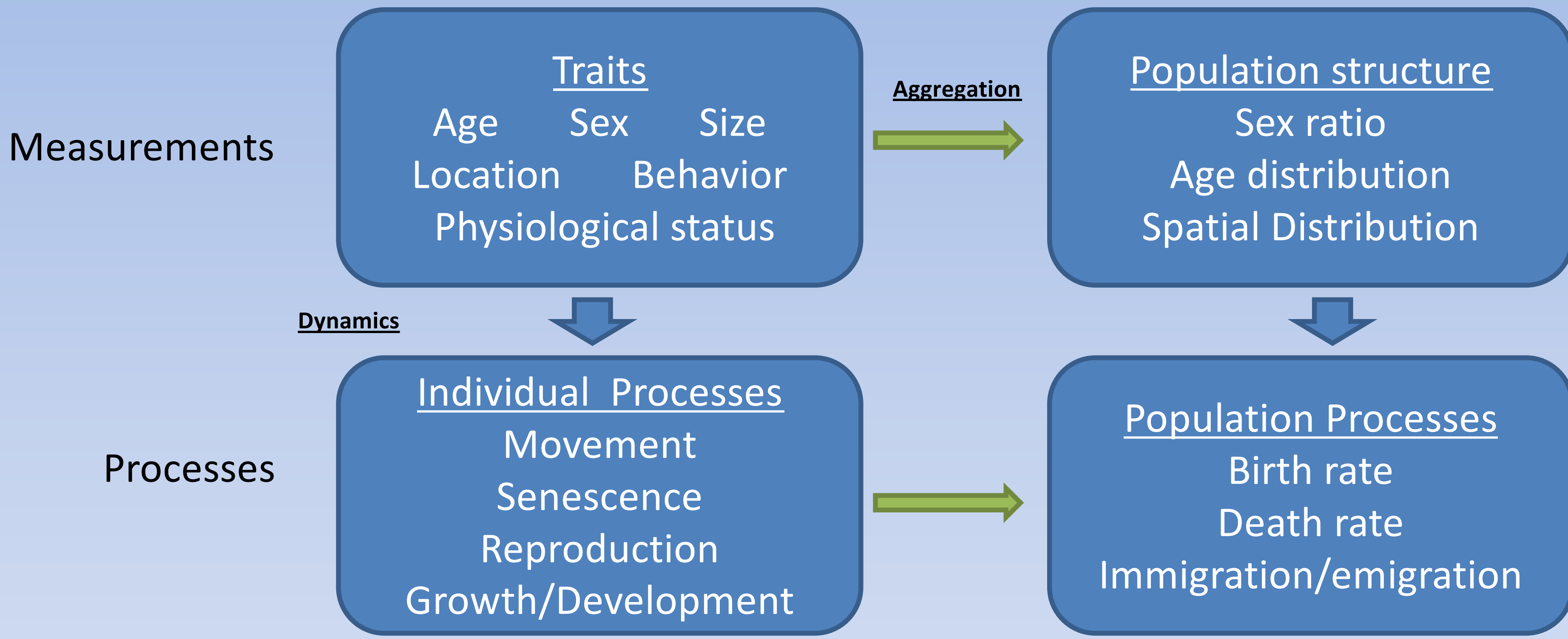
Continuous
$$\frac{dN}{dt} = rN(t)$$

Logistic Growth:

Discrete
$$N_{t+1} = N_t\left(1 + R\left(1 - \frac{N}{K}\right)\right)$$

Continuous
$$\frac{dN}{dt} = rN\left(\frac{K - N}{N}\right)$$

Objection: Measurement shows individuals to be different



Individual variation well documented and the basis for developments of biological theory, i.e., evolution by natural selection

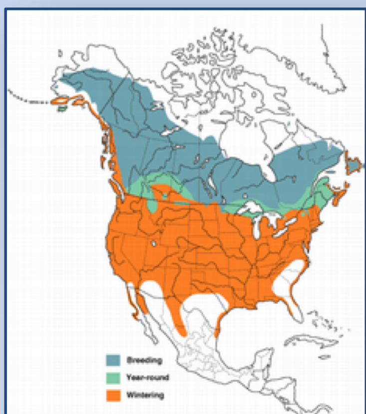
Sources of individual heterogeneity

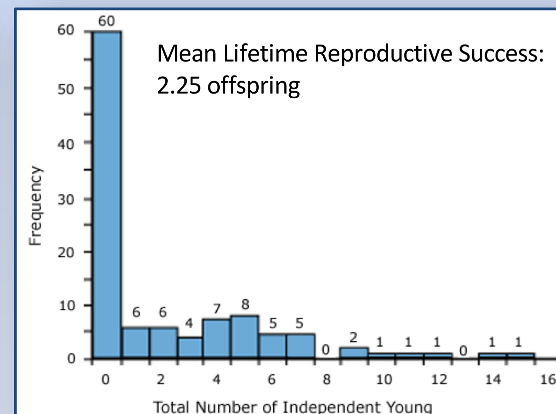
- Heritable genetic variation
- Environmental variation
- Maternal effects
- Mutation

Heterogeneity & Stochasticity

- *Heterogeneity* refers to *differences* among individuals
- *Stochasticity* expresses the idea that variation is *randomly distributed*

Strategy: Begin by assuming all variation “un-attributable” (demographic stochasticity) and build in heterogeneity later





http://bna.birds.cornell.edu

Simple Birth-Death Process

- Denote per capita birth rate by *b* and per capita death rate by *d*
- In a small interval of time h, the probability that a given individual reproduces is the product p(birth)=*bh* and the probability that it dies is p(death)=*dh*
- From this, we can write down the following equation

$$p_n(t+h) = p_n(t)(1-n(b+d)h) + p_{n-1}(n-1)bh + p_{n+1}(n+1)dh$$

- Dividing by *h* and letting *h*→0 yields the following differential equation

$$\frac{dp_n}{dt} = b(n-1)p_{n-1} - (b+d)np_n + d(n+1)p_{n+1}$$

- This differential equation can be solved using the method of *generating functions*
- If we start with n_0=1

$$p_0(t) = \frac{d - de^{-(b-d)t}}{b - de^{-(b-d)t}}$$

Simple Birth-Death Process

• Since the dynamics are density independent, the probability of extinction for n0 individuals is just the joint probability of extinction of their independent lineages

$$p_0(t)=\left(\frac{d-de^{-(b-d)t}}{b-de^{-(b-d)t}}\right)^{n_0}$$

• For *b<d* , we can rewrite this equation as

$$p_0(t)=\left(\frac{de^{(b-d)t}-d}{be^{(b-d)t}-d}\right)^{n_0}$$

•Taking *t*→∞, we have the *ultimate probability of extinction*

$$p_0(t)=\left(\frac{de^{(b-d)t}-d}{be^{(b-d)t}-d}\right)^{n_0}\rightarrow\left(\frac{-d}{-d}\right)^{n_0}=1$$

Simple Birth-Death Process

- For $b>d$ and $t \to \infty$, the ultimate probability of extinction is

$$p_0(t) = \left(\frac{d - de^{-(b-d)t}}{b - de^{-(b-d)t}} \right)^{n_0} \to \left(\frac{d}{b} \right)^{n_0}$$

- Further, for all times, we have the mean population size

$$m_n(t) = n_0 e^{(b-d)t}$$

And variance in population size

$$v_n(t) = n_0 \left(\frac{b+d}{b-d} \right) e^{(b-d)t} \left(e^{(b-d)t} - 1 \right)$$

Simple Birth-Death Process

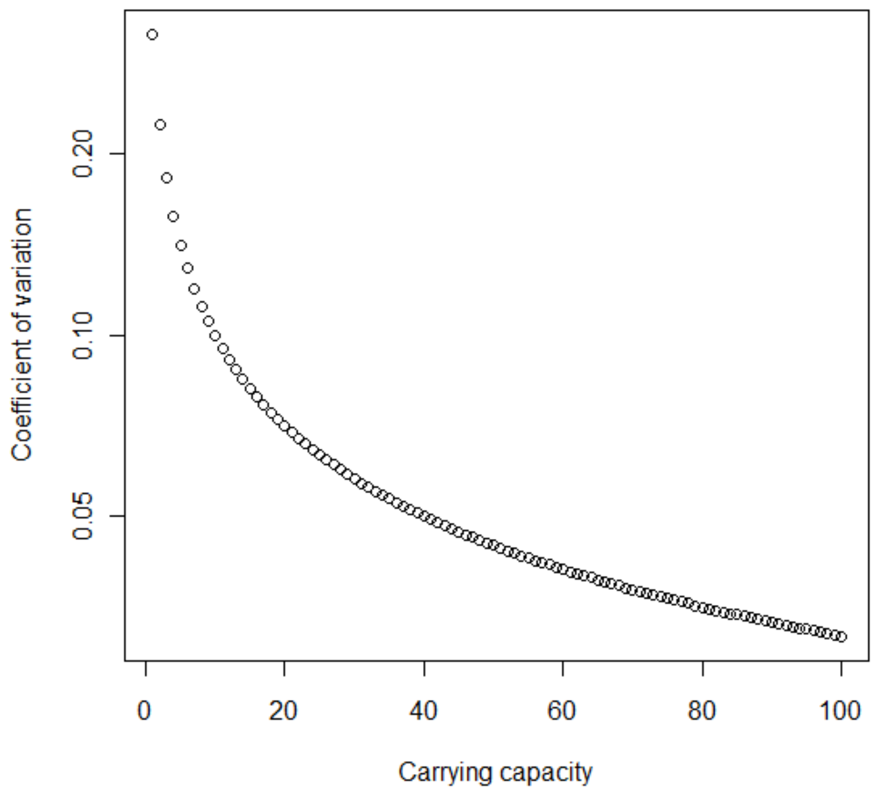
- The coefficient of variation sqrt(v(t))/m(t) is thus

$$cv_n(t) = (b+d)^{1/2}(b-d)^{-1/2} n_0^{-1/2}$$

An example of the square root rule!

Square root scaling rule

Square root scaling rule of demographic stochasticity: Demographic stochasticity is a kind of sampling error and gives rise to a relation between population size and its coefficient of variation over time such that the coefficient of variation is proportional to the inverse square root of the average population size.



Demographic stochasticity will have bigger impacts on small populations

Demographic stochasticity: Discrete time example

b = 0.4 (on average, each individual produces 0.4 offspring per year)

s = 0.6 (each individual has an annual survivorship probability of 0.6, implying that on average 60% of the population will survive from one year to the next)

$$N_{t+1} = N_t + bN_t - dN_t$$

	Pop1	Pop2
Nt =	10	25

Demographic stochasticity example

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	Pop1	Pop2
Nt =	10	25

Ind.	Random	
1	0.5938	no offspring
2	0.2348	one offspring
3	0.4284	no offspring
4	0.7927	no offspring
5	0.3123	one offspring…

Demographic stochasticity example

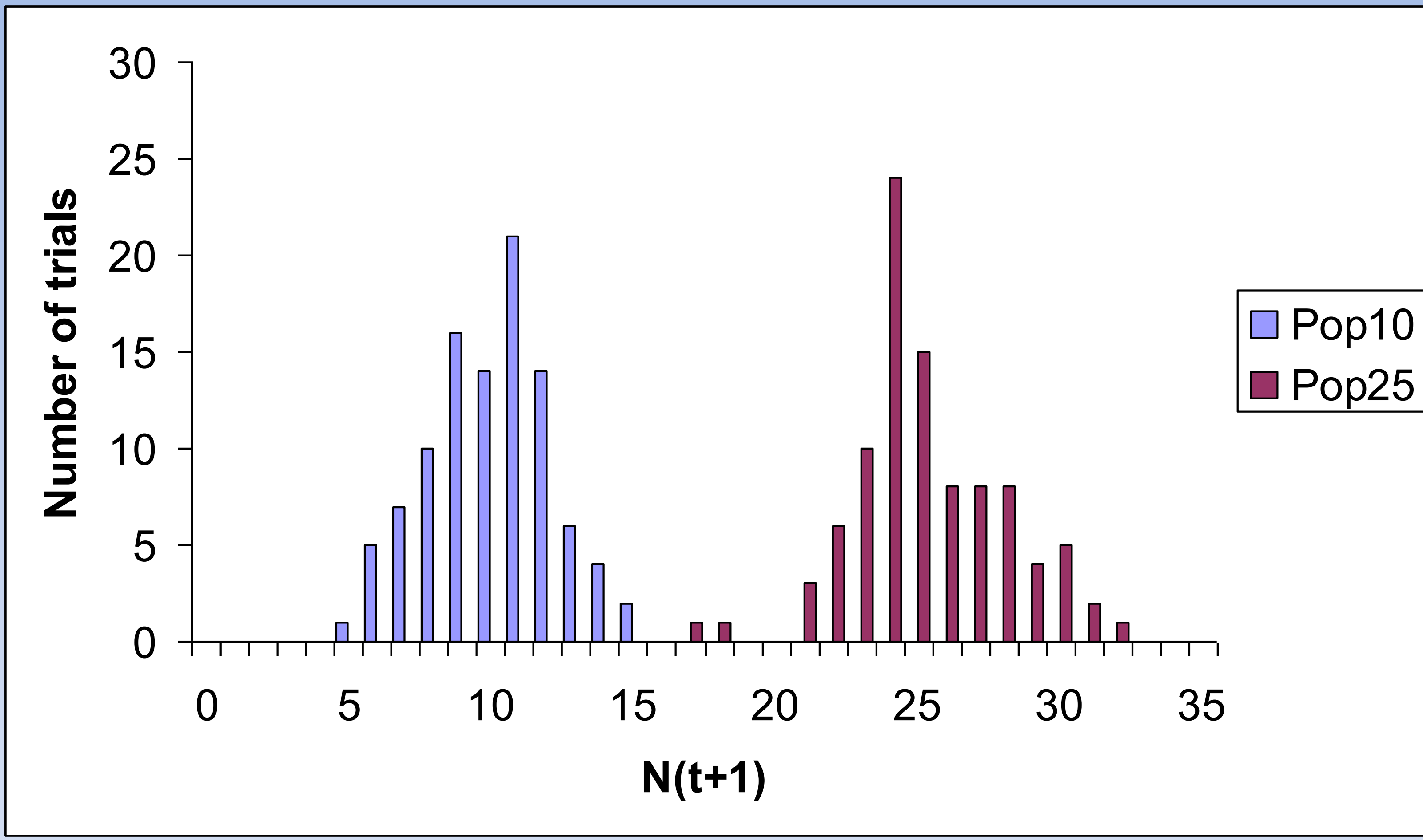
$$N_{t+1} = N_t + bN_t - dN_t$$

		Pop1	Pop2	
Nt	=	10	25	
Nt+1	=	10	25	deterministic

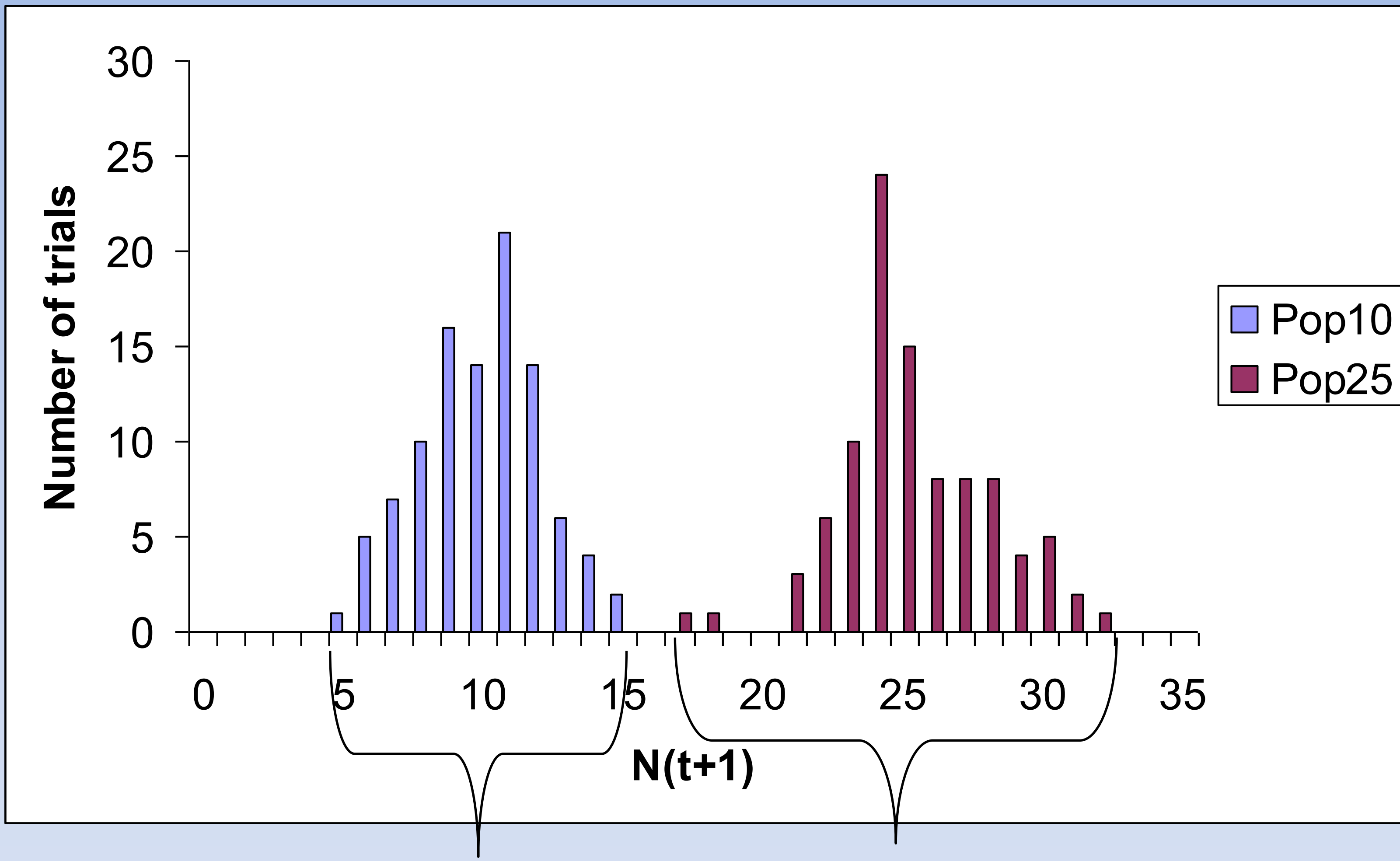
Remember *death rate* = 1 – *s* = 1 – 0.6 = 0.4

So b-d=0

Demographic stochasticity example



Demographic stochasticity example



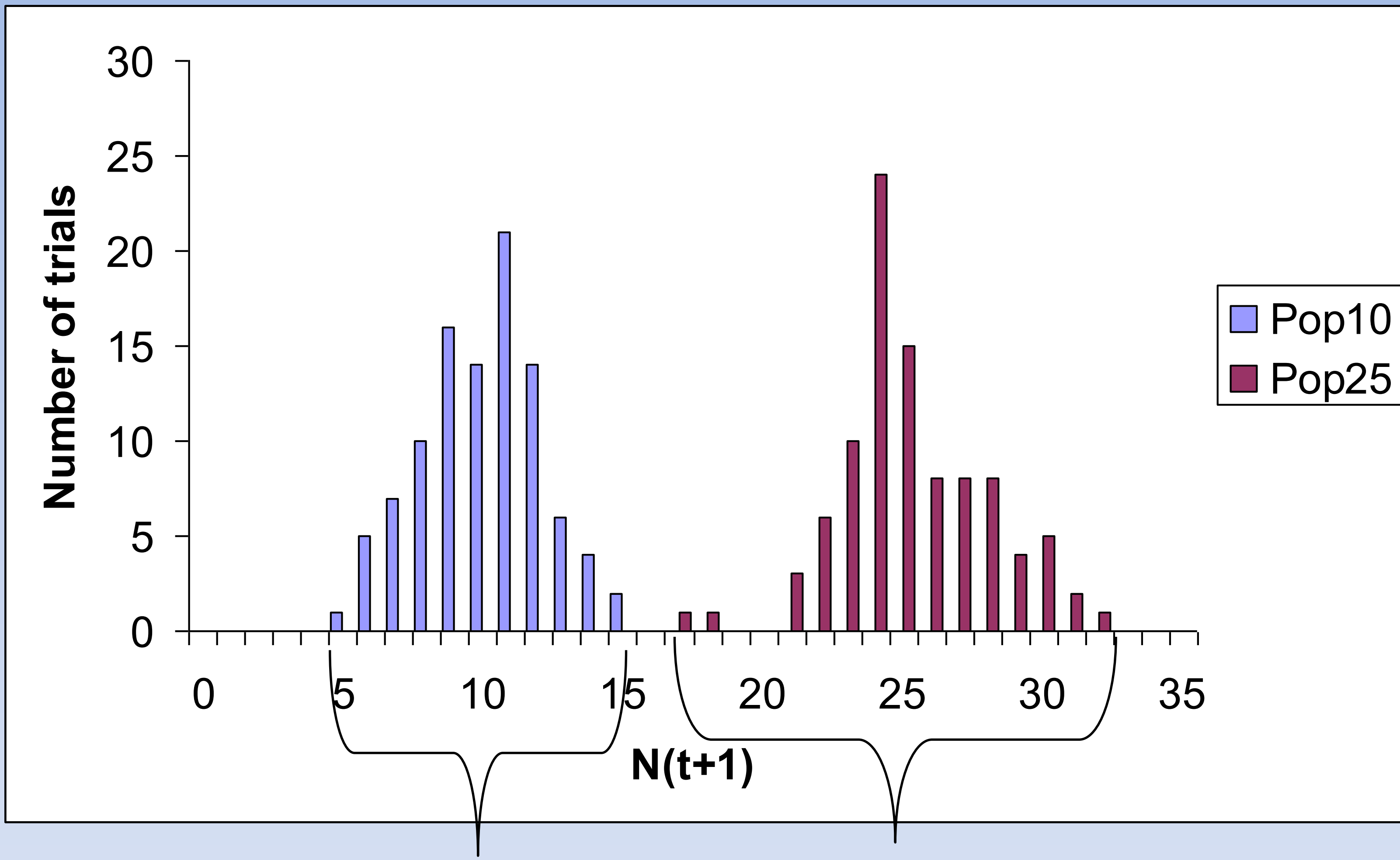
5-15

(SD=2.17)

17-32

(SD=2.84)

Demographic stochasticity example



5-15

(SD=2.17)

CV=0.22

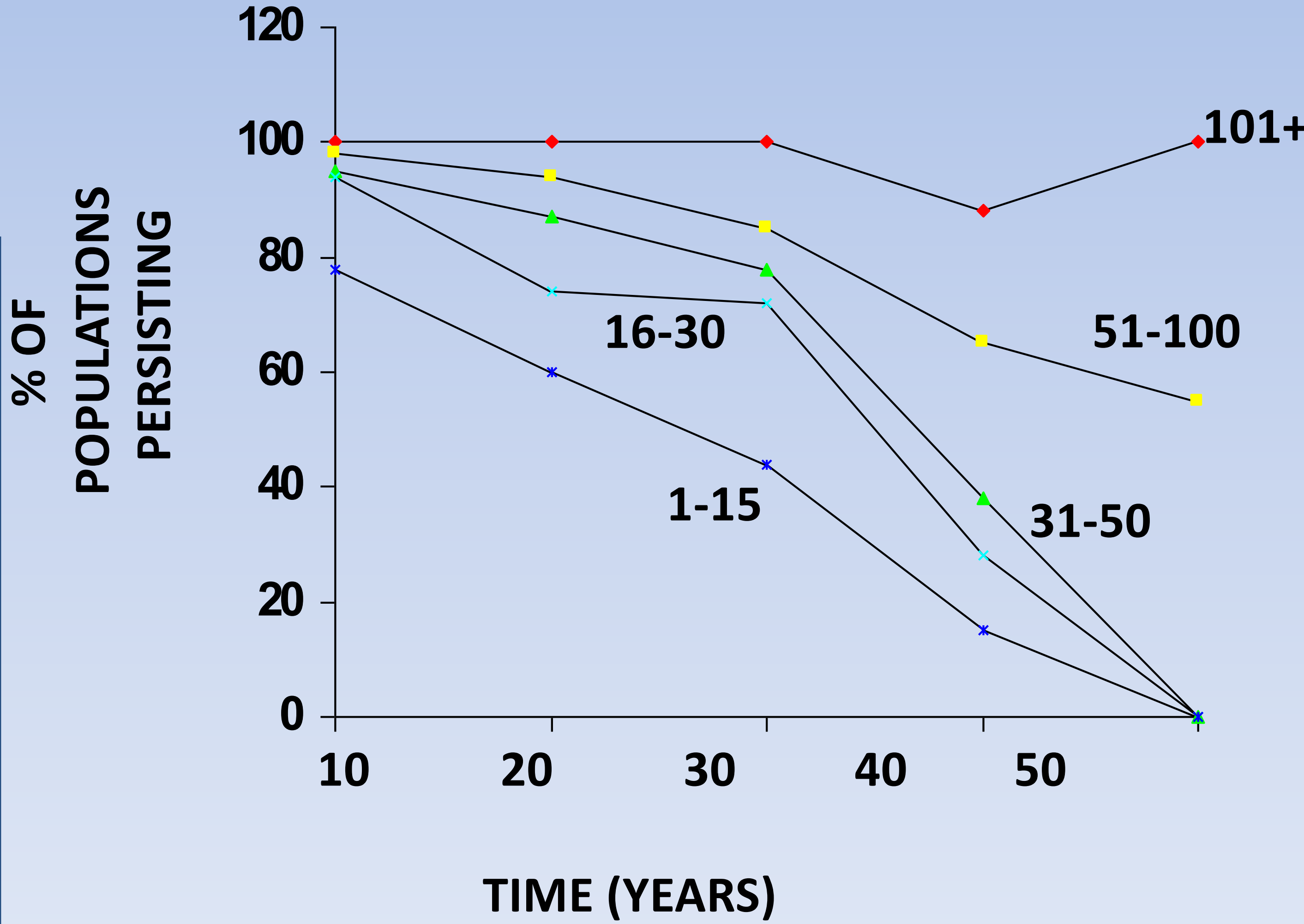
17-32

(SD=2.84)

CV=0.11

Real World Example: Bighorn Sheep

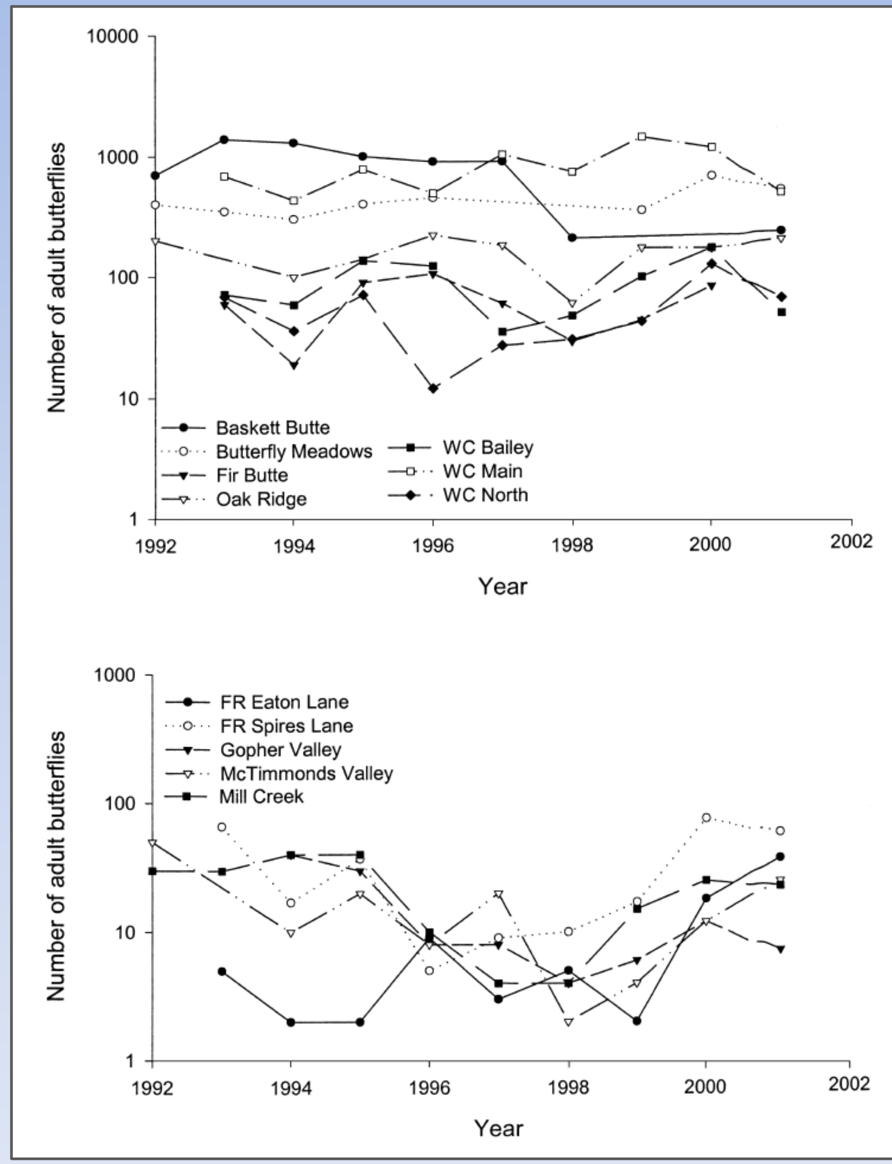
Berger (1990) *Conservation Biology*



Population viability analysis

Population viability analysis is an approach to risk analysis for species extinction that focuses on the potential outcomes of a stochastic population growth process.

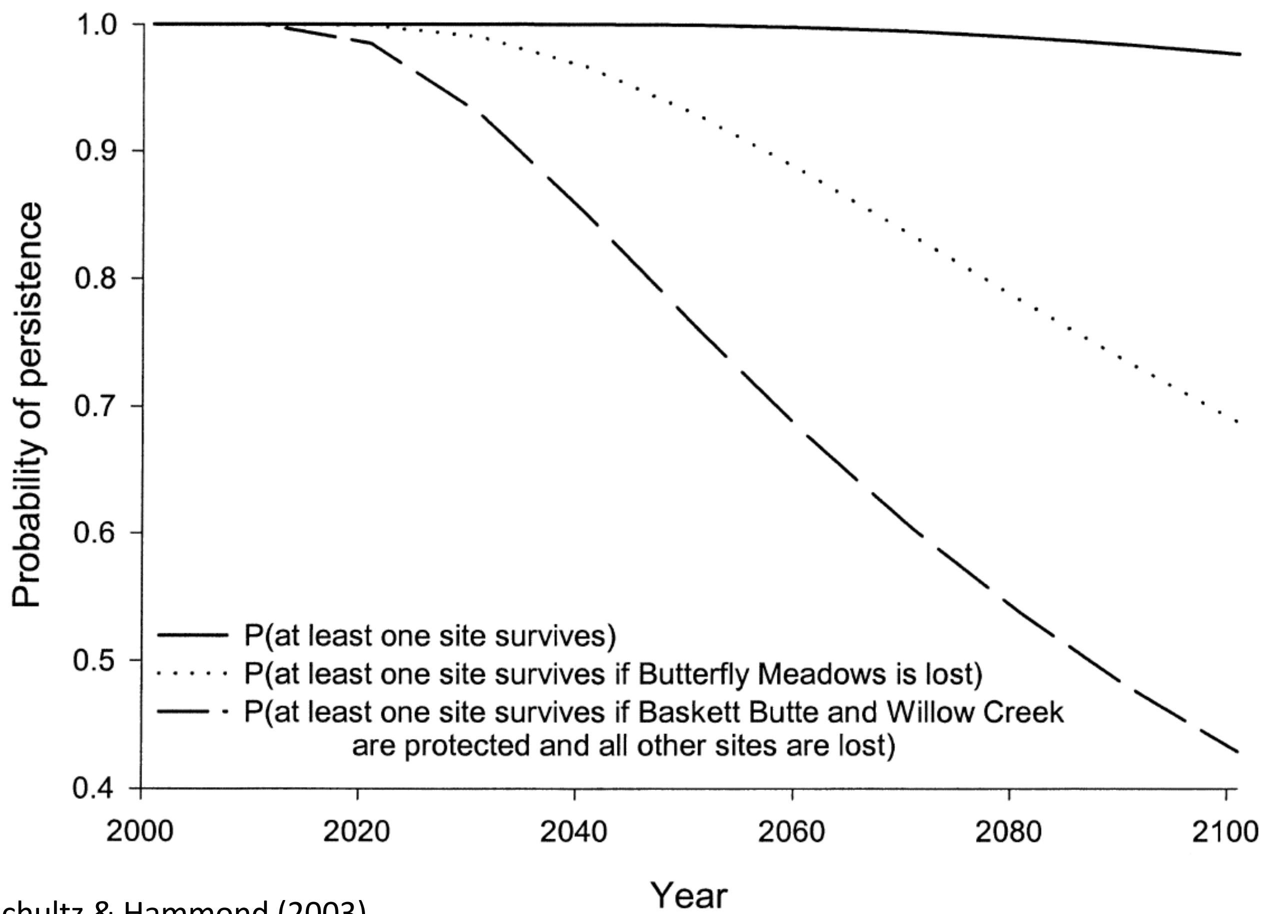
Fender's Blue Butterfly (*Icaricia icarioides fenderi*)



Schultz & Hammond (2003)

Extinction probability and IUCN Criterion E

Criterion E. Quantitative analysis showing the probability of extinction in the wild is at least 20% within 20 years or five generations, whichever is the longer.



Schultz & Hammond (2003)

Summary

- Individuals vary in ways that affect fitness
  - Morphology, age, behavior, sex, location
- Unattributed variation gives rise to demographic stochasticity
- Demographic stochasticity gives rise to the possibility of extinction (even if mean b > mean d)
- Small populations more affected by demographic stochasticity (fewer samples of birth and death rates means less chance to average out)
- These theories are used for population viability analysis and IUCN criteria
- ***HOMEWORK: Extinction chapter, Q2, due 5pm on September 6th***