\[ \frac{dn}{dt} = (b - m)n \]
\[ = bn - mn \]

\[ \text{sub } b = \frac{rR}{k+R} \]

\[ \frac{dn}{dt} = \frac{rRN}{k+R} - mn \]

at eqbm: \[ \emptyset = \frac{rR^*}{k+R^*} - mn \]

\[ \frac{rR^*}{k+R^*} = m \]

\[ rR^* = m(k+R^*) \]

\[ rR^* - mR^* = mk \]

\[ R^*(r - m) = mk \]

\[ R^* = \frac{mk}{r - m} \]
2. \( R^* A. \text{formosa} \approx 1 \)

\( R^* S. \text{ulna} < 1 \)

3. A. formosa excluded (higher \( R^* \))
Lotka-Volterra competition question: In eastern Africa, lions and hyenas are in competition with one another. For the lion, $K_1 = 75$ and for the hyena, $K_2 = 300$. Competition parameters are $a_{12} = 2.5$ and $a_{21} = 3.5$ (where $a_{12}$ represents the effect of species 2 on species 1). Suppose the initial population sizes at a site are 30 lions and 75 hyenas. Plotting lion abundance on the $x$-axis, and hyena abundance on the $y$-axis, plot the isoclines (a.k.a. nullclines, ZNGIs) for each species, and plot these initial population sizes paying attention to details such as intercepts on the $x$ and $y$-axis and accurate placement of initial conditions. Predict the short-term dynamics of each population and the final outcome of interspecific competition.

\[
\frac{dN_1}{dt} = r_1 N_1 \left( \frac{k_1 - (N_1 + a_{12} N_2)}{k_1} \right)
\]

So, the $N_1$ coexistence nullcline is given by $k_1 - (N_1 + a_{12} N_2) = 0$.

\[k_1 = N_1 + a_{12} N_2\]
\[a_{12} N_2 = k_1 - N_1\]
\[N_2 = \frac{k_1 - N_1}{a_{12}}\]

Similarly, for $N_2$:

\[N_2 = k_2 - a_{21} N_1\]

For $k_1 = 75$, $k_2 = 300$, $a_{12} = 2.5$, $a_{21} = 3.5$ then:

\[N_1 \text{ y-intercept } = \frac{k_1}{a_{12}} = \frac{75}{2.5} = 30\]
\[N_2 \text{ y-intercept } = k_2 = 300\]
\[N_1 \text{ x-intercept } = k_1 = 75\]
\[N_2 \text{ x-intercept } = \frac{k_2}{a_{21}} = \frac{300}{3.5} = 85.7\]

Short term:

- $N_1$ decreases, $N_2$ increases.

Long term:

- $N_1$ goes extinct
- $N_2$ goes to carrying capacity $K_2$
\textit{Homework}

1. Use equations 2 to derive the coexistence equilibrium shown in equation 3.

2. Sketch an example of a predator-prey interaction exhibiting damped oscillations as both a regular plot (x-axis=time, y-axis=both population sizes) and a phase portrait. Label the initial conditions and any stable equilibria.

3. Write down a model with both predator and prey density-dependent growth, a type 2 functional response and predator immigration.

4. Draw coexistence (non-zero) nullclines for the 3 models given by equations 1, 2 and 13. Which models have density-dependence? What is the relationship between nullclines and density-dependence/independence?

5. By calculating diagonal elements of the Jacobian for the model represented by equations 2 show that the Trace for the coexistence equilibrium is $-e/bcK$.

\begin{equation}
\text{Solve simultaneous equations:} \quad \frac{r}{n} \left(1 - \frac{n}{K}\right) - bnp = 0 \quad (i) \\
bcnp - ep = 0 \quad (ii)
\end{equation}

From (ii) either $\hat{p} = 0$ or $bc\hat{n} = e$ , but $\hat{p} = 0$ \textit{is not part of a coexistence equilibrium}.

Substitute $\hat{n} = e/bc$ into (i), first noting that, from (i) either $\hat{n} = 0$ or $r\left(1 - \frac{\hat{n}}{K}\right) - b\hat{p} = 0$ , but $\hat{n} = 0$ \textit{is not part of a coexistence equilibrium}.

This rearranges to

\begin{equation}
\hat{p} = \frac{r}{b} \left(1 - \frac{\hat{n}}{K}\right) = \frac{r}{b} \left(1 - \frac{e}{bc}\right)
\end{equation}

Coexistence equilibrium: $\hat{n} = \frac{e}{bc}$ , $\hat{p} = \frac{r}{b} \left(1 - \frac{e}{bc}\right)$.
The diagram and equations illustrate predator-prey dynamics. The equations are:

\[ \frac{dn}{dt} = r n \left(1 - \frac{n}{K}\right) - a np \frac{n}{aT_n + 1} \]

\[ \frac{dp}{dt} = \frac{b a p n}{aT_n + 1} \left(1 - \frac{f}{k}\right) + \varepsilon \]

The text also mentions:

- Initial conditions
- Time
- Equilibrium density
- No density dependence
- Immigration

The diagram visualizes the interactions between prey and predator populations over time.
\(\frac{dn}{dt} = r n (1 - \frac{n}{K}) - b n p = f\)

\(\frac{dp}{dt} = b c n p - e p = g\)

Jacobian \(J = \begin{bmatrix} \frac{\partial f}{\partial n} & \frac{\partial f}{\partial p} \\ \frac{\partial g}{\partial n} & \frac{\partial g}{\partial p} \end{bmatrix} = \begin{bmatrix} \frac{r - 2 r n - b p}{K} - b n \\ b c p & b c n - e \end{bmatrix}\)

Trace \(J = r - \frac{2 r n}{K} - b p + b c n - e\)

When \(n = \frac{a}{b c}\), Trace \(J = r - 2 e r f - b r + b c e - e\)

& \(p = \frac{r}{b}\) \(\int \frac{1}{b c k} = -\frac{2 r e}{b c k}\)