(1) $\frac{dn}{dt} = (b - m)n$

$= bn - mn$

sub $b = \frac{rR}{k+R}$ {monod equation}

$\frac{dn}{dt} = \frac{rRn}{k+R} - mn$

at eqbm: $\emptyset = \frac{rR^*n}{k+R^*} - mn$

$\frac{rR^*}{k+R^*} = m$

$rR^* = m(k+R^*)$

$rR^* - mR^* = mk$

$R^*(r-m) = mk$

$R^* = \frac{mk}{r-m}$

(2) R^* A. formosa ~ 1

R^* S. Ulna < 1

(3) A. formosa excluded (higher R^*)

Lotka-Volterra competition question: In eastern Africa, lions and hyenas are in competition with one another. For the lion, $K_1 = 75$ and for the hyena, $K_2 = 300$. Competition parameters are $\alpha_{12} = 2.5$ and $\alpha_{21} = 3.5$ (where α_{12} represents the effect of species 2 on species 1). Suppose the initial population sizes at a site are 30 lions and 75 hyenas. Plotting lion abundance on the x-axis, and hyena abundance on the y-axis, plot the isoclines (*a.k.a.* nullclines, ZNGIs) for each species, and plot these initial population sizes paying attention to details such as intercepts on the *x* and *y*-axis and accurate placement of initial conditions. Predict the short-term dynamics of each population and the final outcome of interspecific competition.

$$\frac{dN_1}{dt} = r_1 N_1 \left(\frac{K_1 - (N_1 + \alpha_{12} N_2)}{K_1} \right)$$

So, the N_1 coexistence nullcline is given by $K_1 - (N_1 + \alpha_{12} N_2) = \emptyset$

ie, $K_1 = N_1 + \alpha_{12} N_2$

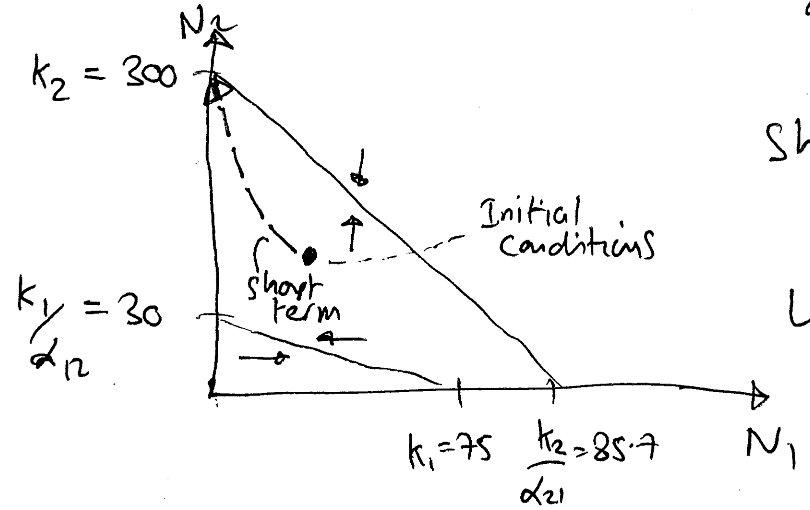
$\alpha_{12} N_2 = K_1 - N_1$, Similarly, for N_2:

$$N_2 = \frac{K_1}{\alpha_{12}} - \frac{N_1}{\alpha_{12}} \qquad N_2 = K_2 - \alpha_{21} N_1$$

For $K_1 = 75$, $K_2 = 300$, $\alpha_{12} = 2.5$, $\alpha_{21} = 3.5$ then:

N_1 yintercept $= \frac{K_1}{\alpha_{12}} = \frac{75}{2.5} = 30$, N_2 yintercept $= K_2 = 300$

N_1 xintercept $= K_1 = 75$, N_2 xintercept $= \frac{K_2}{\alpha_{21}} = \frac{300}{3.5} = 85.7$



Short term:

N_1 decreases, N_2 increases.

Longterm:

N_1 goes extinct

N_2 goes to carrying capacity K_2

Homework

1. Use equations 2 to derive the coexistence equilibrium shown in equation 3.

2. Sketch an example of a predator-prey interaction exhibiting damped oscillations as both a regular plot (x-axis=time, y-axis=both population sizes) and a phase portrait. Label the initial conditions and any stable equilibria.

3. Write down a model with both predator and prey density-dependent growth, a type 2 functional response and predator immigration.

4. Draw coexistence (non-zero) nullclines for the 3 models given by equations 1, 2 and 13. Which models have density-dependence? What is the relationship between nullclines and density-dependence/independence?

5. By calculating diagonal elements of the Jacobian for the model represented by equations 2 show that the Trace for the coexistence equilibrium is $-e/bcK$.

① Solve simultaneous equations $rn\left(1-\frac{n}{K}\right) - bnp = \emptyset$ (i)

$bcnp - ep = \emptyset$ (ii)

From (ii) either $\hat{p} = \emptyset$ or $bc\hat{n} = e$ ie/ $\hat{n} = \frac{e}{bc}$, but $\hat{p} = \emptyset$ is not part of a coexistence equilibrium

Substitute $\hat{n} = e/bc$ into (i), first noting that, from (i) either

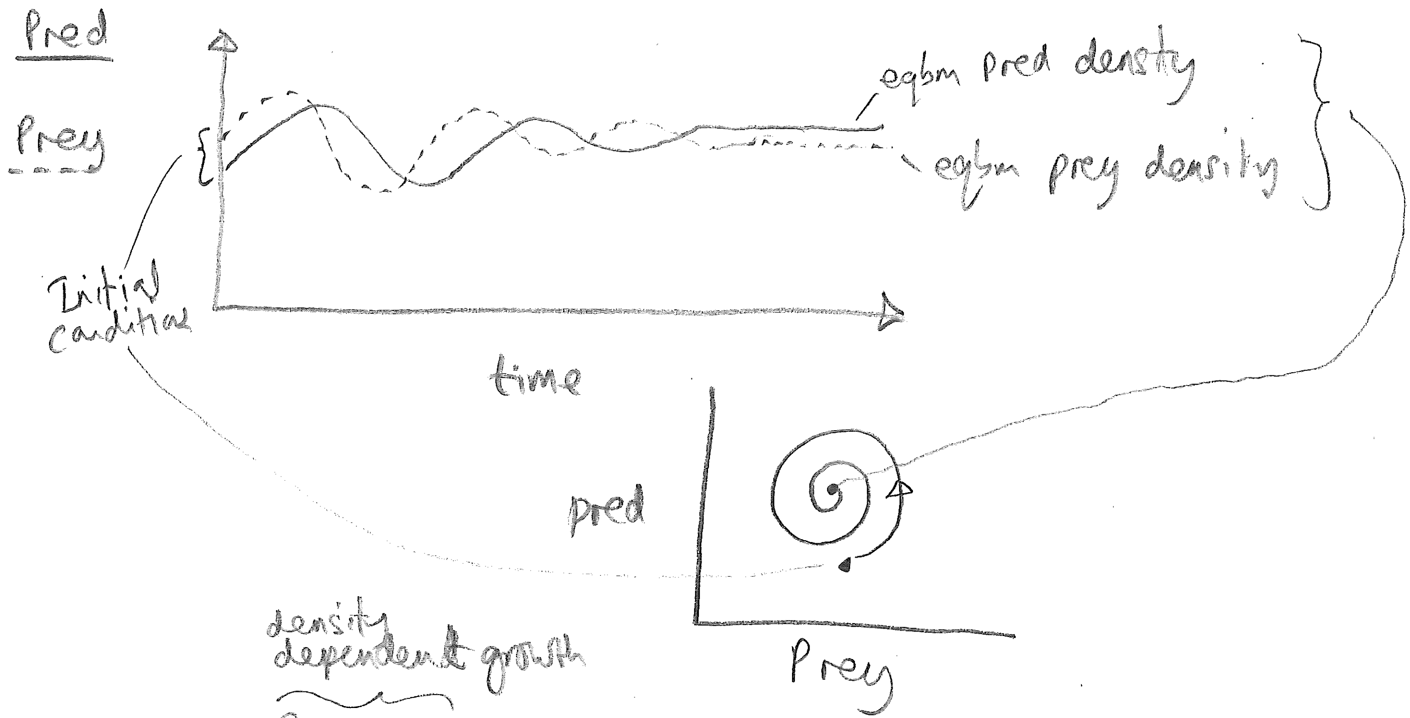
$\hat{n} = \emptyset$ or $\underbrace{r\left(1-\frac{\hat{n}}{K}\right) - b\hat{p} = \emptyset}$, but $\hat{n} = \emptyset$ is not part of a coexistence equilibrium

This rearranges to

$$\hat{p} = \frac{r}{b}\left(1-\frac{\hat{n}}{K}\right) = \frac{r}{b}\left(1-\frac{\hat{n}}{K}\right)$$

Coexistence equilibrium $\hat{n} = \frac{e}{bc}$, $\hat{p} = \frac{r}{b}\left(1-\frac{e}{bcK}\right)$

②



③

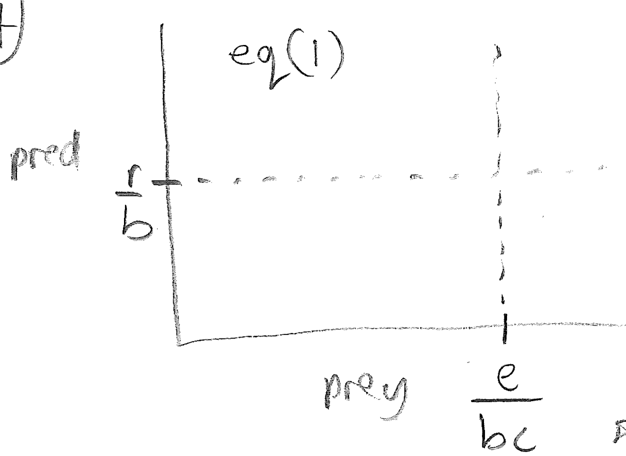
$$\frac{dn}{dt} = \underbrace{rn\left(1-\frac{n}{K}\right)}_{\text{density dependent growth}} - \frac{anp}{aT_h n + 1} \quad \text{type II}$$

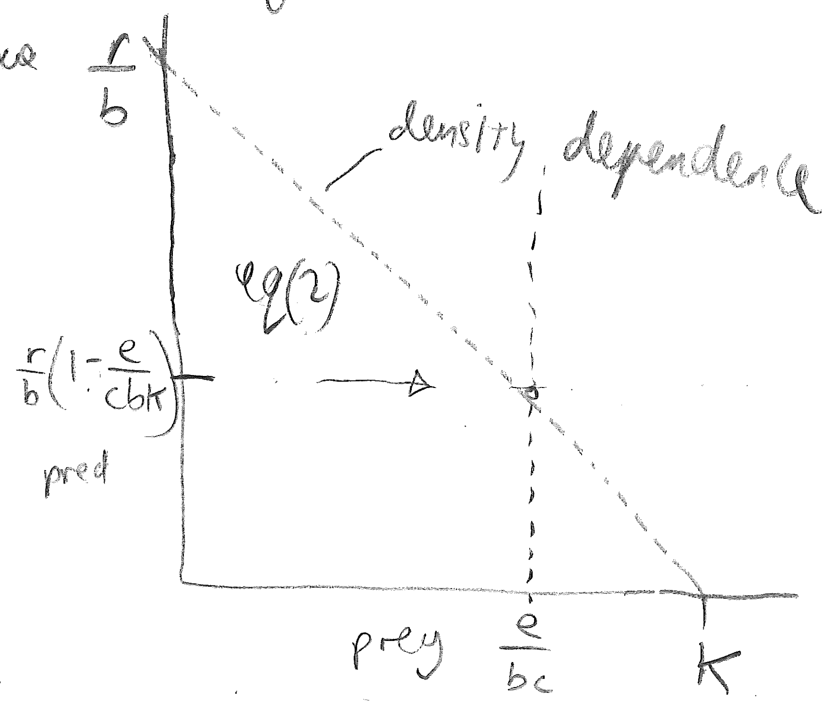
$$\frac{dp}{dt} = \frac{banp}{aT_h n + 1}\left(1-\frac{p}{K_p}\right) + \varepsilon$$

density dependence

immigration

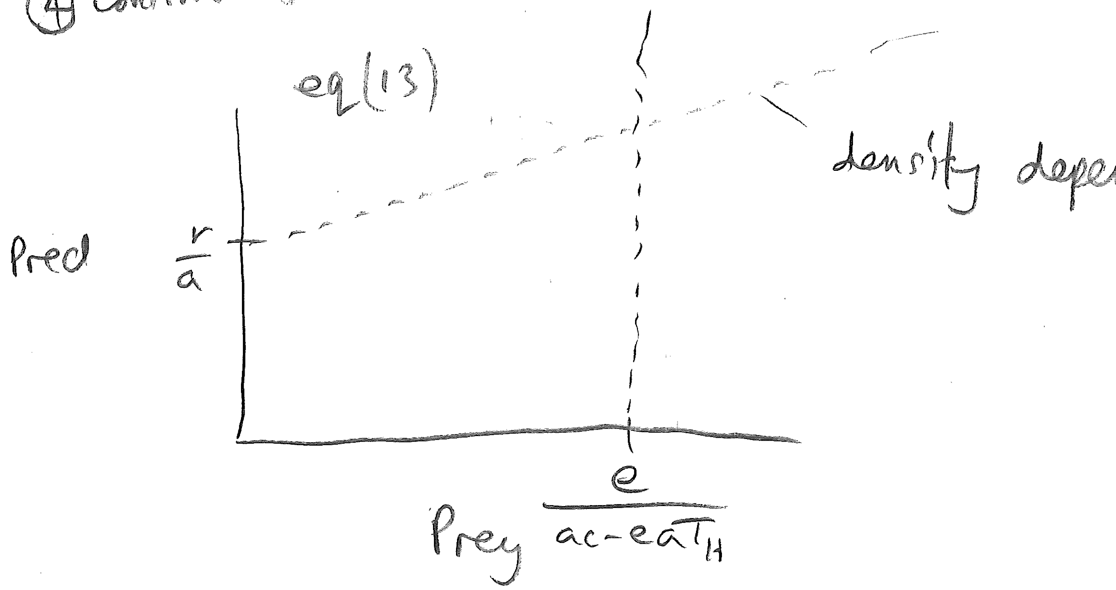
④





no density dependence

④ continued



⑤ $$\frac{dn}{dt} = rn\left(1 - \frac{n}{k}\right) - bnp = f$$

$$\frac{dp}{dt} = bcnp - ep = g$$

Jacobian $$J = \begin{bmatrix} \frac{\partial f}{\partial n} & \frac{\partial f}{\partial p} \\ \frac{\partial g}{\partial n} & \frac{\partial g}{\partial p} \end{bmatrix} = \begin{bmatrix} r - \frac{2rn}{k} - bp & -bn \\ bcp & bcn - e \end{bmatrix}$$

$$\text{Trace}(J) = r - \frac{2rn}{k} - bp + bcn - e$$

when $\left.\begin{matrix} n = \frac{e}{bc} \\ \& \; p = \frac{r}{b} \end{matrix}\right\}$, $$\text{Trace}(J) = r - \frac{2re}{bck} - \frac{br}{b} + \frac{bce}{bc} - e$$

$$= -\frac{2re}{bck}$$