# Population Growth \& Decline 

## ECOL 4000/6000

What is the difference between an open and closed population?


## Decline and growth of Yellowstone Grizzly Bears



## Decline and growth of Yellowstone Grizzly Bears




Ursus arctos horribilis

## Decline and growth of Yellowstone Grizzly Bears



A population ecology goal: to understand population-level patterns and be able ultimately to predict changes in them


## Questions about growth and decline



- How much change in mortality was required to shift the balance from decline to growth?
- If interventions had not been undertaken, would the population have gone extinct? When?
- Is the population safe now? What is the chance that it might still go extinct?


## What causes growth and decline?

## What causes growth and decline?

Four key demographic processes

- Reproduction (+)
"Open" Population
- Mortality/Survivorship (-)
- Immigration (+)
- Emigration (-)


## What causes growth and decline?

Four key demographic processes

- Reproduction (+)
"Closed" Population
- Mortality/Survivorship (-)
- Immigration (+)
- Emigration (-)


## Representing demography

The fundamental equation of population ecology

$$
\Delta N=N_{t+1}-N_{t}=B_{N}-D_{N}+I_{N}-E_{N}
$$

## Representing demography

The fundamental equation of population ecology

$$
\Delta N=N_{t+1}-N_{t}=B_{N}-D_{N}+I_{N}-E_{N}
$$

- Assume a closed population (ignore I \& E)
- Assume no heterogeneity
- Assume no density dependence


## Representing demography

The fundamental equation of population ecology

$$
\Delta N=N_{t+1}-N_{t}=B_{N}-D_{N}+I_{N}-E_{N}
$$

- Assume a closed population
- Assume no heterogeneity
- Assume no density dependence
- Recall that $\mathrm{B}_{\mathrm{N}}$ and $\mathrm{D}_{\mathrm{N}}$ are the number of births and deaths in a population of
size $N$
-If each individual gives
birth to B offspring then
$B_{N}=B N_{t}$
- Similarly, if each individual
has a chance of dying of $D$,
the number of deaths is
$\mathrm{D}_{\mathrm{N}}=\mathrm{DN}_{\mathrm{t}}$
-Define $\lambda=B-D+1$


## Representing demography

The fundamental equation of population ecology

$$
\Delta N=N_{t+1}-N_{t}=B_{N}-D_{N}+I_{N}-E_{N}
$$

- Assume a closed population
- Assume no heterogeneity
- Assume no density dependence

$$
\begin{aligned}
& \Delta N=N_{t+1}-N_{t}=B N_{t}-D N_{t} \\
& N_{t+1}=B N_{t}-D N_{t}+N_{t}=\lambda N_{t}
\end{aligned}
$$

-Recall that $\mathrm{B}_{\mathrm{N}}$ and $\mathrm{D}_{\mathrm{N}}$ are the number of births and deaths in a population of
size N
-If each individual gives birth to B offspring then $B_{N}=B N_{t}$

- Similarly, if each individual
has a chance of dying of $D$,
the number of deaths is
$\mathrm{D}_{\mathrm{N}}=\mathrm{DN}_{\mathrm{t}}$
-Define $\lambda=B-D+1$


## Generalize for more than one time step

$$
N_{t+1}=(B-D+1) N_{t}=\lambda N_{t}
$$

## Generalize for more than one time step

$$
\begin{aligned}
& N_{t+1}=(B-D+1) N_{t}=\lambda N_{t} \\
& N_{t+2}=(B-D+1) N_{t+1}=\lambda N_{t+1}
\end{aligned}
$$

## Generalize for more than one time step

$$
\begin{gathered}
N_{t+1}=(B-D+1) N_{t}=\lambda N_{t} \\
N_{t+2}=(B-D+1) N_{t+1}=\lambda N_{t+1} \\
N_{t+2}=(B-D+1) \lambda N_{t}=\lambda \lambda N_{t}=\lambda^{2} N_{t}
\end{gathered}
$$

## Generalize for more than one time step

$$
\begin{gathered}
N_{t+1}=(B-D+1) N_{t}=\lambda N_{t} \\
N_{t+2}=(B-D+1) N_{t+1}=\lambda N_{t+1} \\
N_{t+2}=(B-D+1) \lambda N_{t}=\lambda \lambda N_{t}=\lambda^{2} N_{t}
\end{gathered}
$$

In general...

$$
N_{t}=\lambda^{t} N_{0}
$$

## Geometric growth and decline



## Estimating $\lambda$ from data

$$
\begin{aligned}
& N_{t}=\lambda^{t} N_{0} \\
& N_{t} / N_{0}=\lambda^{t} \\
& \log \left(N_{t} / N_{0}\right)=\log \left(\lambda^{t}\right) \\
& \log \left(N_{t} / N_{0}\right)=t \log \lambda \\
& \frac{\log \left(N_{t} / N_{0}\right)}{t}=\log \lambda \\
& e^{\frac{\log \left(N_{t} / N_{0}\right)}{t}}=\lambda
\end{aligned}
$$

## Worked Example

Given that the Yellowstone Grizzly population was 44 in 1959 and 34 in 1975, what was the average annual reproductive ratio during this period?

Note: natural log used here

## Estimating $\lambda$ from data

$$
\begin{aligned}
& N_{t}=\lambda^{t} N_{0} \\
& N_{t} / N_{0}=\lambda^{t} \\
& \log \left(N_{t} / N_{0}\right)=\log \left(\lambda^{t}\right) \\
& \log \left(N_{t} / N_{0}\right)=t \log \lambda \\
& \frac{\log \left(N_{t} / N_{0}\right)}{t}=\log \lambda \\
& e^{\frac{\log \left(N_{t} / N_{0}\right)}{t}}=\lambda
\end{aligned}
$$

## Worked Example

Given that the Yellowstone Grizzly population was 44 in 1959 and 34 in 1975, what was the average annual reproductive ratio during this period?

```
\lambda=\mp@subsup{e}{}{\operatorname{log}(34/44//16}=\mp@subsup{e}{}{-0.0161}=0.984
```



Note: natural log used here

## Estimating $\lambda$ from data

$$
\begin{aligned}
& N_{t}=\lambda^{t} N_{0} \\
& N_{t} / N_{0}=\lambda^{t} \\
& \log \left(N_{t} / N_{0}\right)=\log \left(\lambda^{t}\right) \\
& \log \left(N_{t} / N_{0}\right)=t \log \lambda \\
& \frac{\log \left(N_{t} / N_{0}\right)}{t}=\log \lambda \\
& e^{\frac{\log \left(N_{N} / N_{0}\right)}{t}}=\lambda
\end{aligned}
$$

## Question

Notice how the population seemed to turn a corner sometime between 1973 and 1978. If we calculate the average reproductive ratio between one of these years and the end of the time series in 1997, will we find
(a) $\lambda<1$
(b) $\lambda=1$
(c) $\lambda>1$

Does it matter what year we choose to be $t=0$ ? How will this choice affect our estimate of $\lambda$ ?

Note: natural log used here

## Review - so far (discrete time model)

- We see the solution is geometric in time
- Change in population size depends on current size $(N)$ and growth rate $(\lambda)$
- 3 possible outcomes
$\lambda>1$ ( N increases)
$\lambda=1$ ( N remains constant)
$\lambda<1$ ( $N$ declines)
- We can estimate $\lambda$ from time series data


## Continuous time model

- Many species reproduce more-or-less continuously over time
- EXERCISE: work through equations 13-21 in class
- $d n / d t=b^{*} n-d^{*} n \rightarrow n=n_{0}{ }^{*} e^{*}{ }^{*}$


## Continuous time model

- Similar 'threshold' as discrete-time model
- Make sure you follow equations 22-25
- $r=\ln (\lambda)$


| Phenomenon | Discrete time | Continuous time |
| :--- | :--- | :--- |
| Population <br> decline | $\lambda<1$ | $r<0$ |
| Population <br> constant | $\lambda=1$ | $r=0$ |
| Population <br> growth | $\lambda>1$ | $r>0$ |

## Summary and expectations

- Fundamental equation (know this)
- open (including immigration/emigration)
- closed (just births/deaths)
- Discrete time model ( $\lambda=1$ threshold)
- Should be able to get $\lambda=$... from $N_{t}=\lambda^{t} N_{0}$
- Continuous time model ( $r=0$ threshold)
- Should be able to solve dn/dt=r*n
- Can use data to estimate parameters (e.g. $\lambda$ )
- Should be able to do calculation similar to bears worked example
- Can use model + parameters to make predictions
- Should be able to make a straightforward prediction of a future population size using either model, provided with parameters and initial population size


## Homework 1

- All homework questions of chapter 1 (\#1-6)
- Questions can be typed or scanned (incl. photo if clear)
- Study groups encouraged - turn in individual answers
- Email (awpark@uga.edu) by 5pm Aug 30 th

